

# Double and Half-Angle Formulas

## Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u \quad \text{or} \quad 2 \cos^2 u - 1 \quad \text{or} \quad 1 - 2 \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

## Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} \quad \text{or} \quad \frac{\sin u}{1 + \cos u}$$

1. Find the exact value of each trigonometric function.

a)  $\sin 2x$

b)  $\cos 2x$

c)  $\tan 2x$

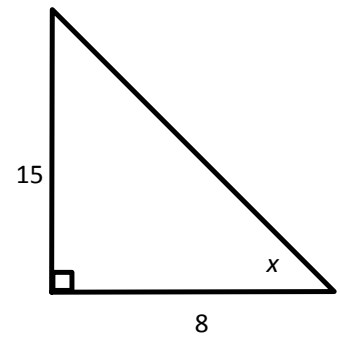
d)  $\csc 2x$

e)  $\sec 2x$

f)  $\cot 2x$

g)  $\sin \frac{x}{2}$

h)  $\cos \frac{x}{2}$



i)  $\tan \frac{x}{2}$

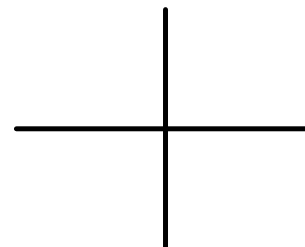
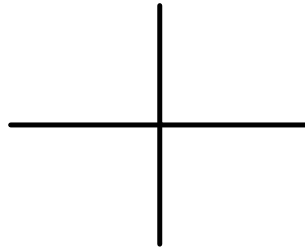
j)  $\csc \frac{x}{2}$

k)  $\sec \frac{x}{2}$

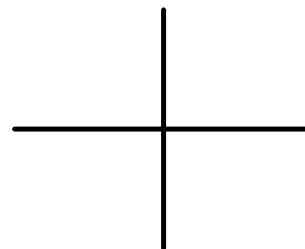
l)  $\cot \frac{x}{2}$

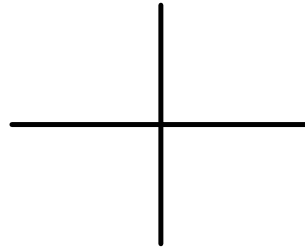
2. Find the solutions to each equation for  $0 \leq x < 2\pi$ .

a)  $\sin 2x - \sin x = 0$



b)  $\cos 2x + \cos x = 0$





3. Find the exact value of each trigonometric function if:

$$\tan u = -\frac{24}{7} \text{ and } \frac{3\pi}{2} < u < 2\pi.$$

a)  $\sin 2u$

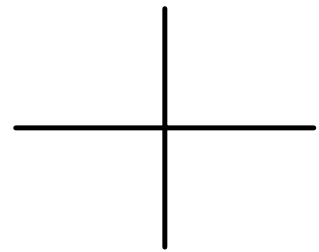
b)  $\cos 2u$

c)  $\tan 2u$

d)  $\sin \frac{u}{2}$

e)  $\cos \frac{u}{2}$

f)  $\tan \frac{u}{2}$



4. Verify each identity.

a)  $\csc 2x = \frac{\csc x}{2 \cos x}$

$$\text{b) } \cos^2 2x - \sin^2 2x = \cos 4x$$

$$\text{c) } \sec \frac{x}{2} = \pm \sqrt{\frac{2 \tan x}{\tan x + \sin x}}$$