Solving Equations by Completing the Square

Standard Form
$y = ax^2 + bx + c$

Vertex Form
$y = a(x-h)^2 + k$

Step 1: Write the *x* terms on the left side of the equation and the constant on the right side of the equation.
Step 2: Divide *b* by 2 and then square it. Add this number to both sides of the equation.
Step 3: Write in vertex form.

1. Solve each equation by completing the square.

a) $x^2 - 6x - 40 = 0$

$+40 \quad +40$

$x^2 - 6x + 9 = 40 + 9$

$b = \frac{6}{2} = (3)^2 = 9$

$(x-3)(x-3) = 49$

$\sqrt{(x-3)^2} = \sqrt{49}$

$x - 3 = \pm 7$

$x - 3 = 7$ $\quad$ $x - 3 = -7$

$+3 \quad +3$ $\quad$ $+3 \quad +3$

$\boxed{x = 10}$ $\quad$ $\boxed{x = -4}$

b) $x^2 + 4x + 3 = 0$

$-3 \quad -3$

$x^2 + 4x + 4 = -3 + 4$

$b = \frac{4}{2} = (2)^2 = 4$

$x^2 + 4x + 4 = 1$

$(x+2)(x+2) = 1$

$\sqrt{(x+2)^2} = \sqrt{1}$

$x + 2 = \pm 1$

$x + 2 = 1$ $\quad$ $x + 2 = -1$

$-2 \quad -2$ $\quad$ $-2 \quad -2$

$\boxed{x = -1}$ $\quad$ $\boxed{x = -3}$

c) $x^2+3x=18$ $+\frac{9}{4}$

$+\frac{9}{4}$

$b=\frac{3}{2}$ $\left(\frac{3}{2}\right)^2=\frac{9}{4}$

$x^2+3x+\frac{9}{4}=\frac{18}{1}\cdot\frac{4}{4}+\frac{9}{4}$

$(x+\frac{3}{2})(x+\frac{3}{2})$ LCD=4

$\frac{72}{4}+\frac{9}{4}=\frac{81}{4}$

$\sqrt{(x+\frac{3}{2})^2}=\sqrt{\frac{81}{4}}$

$x+\frac{3}{2}=\pm\frac{9}{2}$

$x+\frac{3}{2}=\frac{9}{2}$ $x+\frac{3}{2}=-\frac{9}{2}$

$-\frac{3}{2}$ $-\frac{3}{2}$ $-\frac{3}{2}$ $-\frac{3}{2}$

$\boxed{x=3}$ $\boxed{x=-6}$

d) $x^2+4x=1$ $+4$

$+4$

$b=4$ $\frac{4}{2}=(2)^2=4$

$x^2+4x+4=5$

$(x+2)(x+2)=5$

$\sqrt{(x+2)^2}=\sqrt{5}$

$x+2=\pm 2.2$

$x+2=2.2$ $x+2=-2.2$

-2 -2.0 -2 -2.0

$\boxed{x=.2}$ $\boxed{x=-4.2}$

e) $x^2+2x=5$ $+1$

$+1$

$b=2$ $\frac{2}{2}=(1)^2=1$

$x^2+2x+1=6$

$(x+1)(x+1)=6$

$\sqrt{(x+1)^2}=\sqrt{6}$

$x+1=\pm 2.4$

$x+1=2.4$ $x+1=-2.4$

-1 -1.0 -1 -1.0

$\boxed{x=1.4}$ $\boxed{x=-3.4}$

2. Rewrite each quadratic function in vertex form.

$$y = a(x-h)^2 + k$$

a) $y = x^2 - 4x + 8$

$-8 \quad -8$

$y - 8 = x^2 - 4x + 4$

$+4$

$b = 4 \quad \frac{4}{2} = (2)^2 = 4$

$y - 4 = x^2 - 4x + 4$

$y - 4 = (x-2)(x-2)$

$y - 4 = (x-2)^2$

$+4 \quad +4$

$\boxed{y = (x-2)^2 + 4}$

b) $y = -3 + 2x + x^2$

$+3 \quad +3$

$y + 3 = x^2 + 2x + 1$

$+1$

$b = 2 \quad \frac{2}{2} = (1)^2 = 1$

$y + 4 = x^2 + 2x + 1$

$y + 4 = (x+1)(x+1)$

$y + 4 = (x+1)^2$

$-4 \quad -4$

$\boxed{y = (x+1)^2 - 4}$

c) $4y = 4x^2 + 20x - 4$

$\frac{4y}{4} = \frac{4x^2}{4} + \frac{20x}{4} - \frac{4}{4}$

$y = x^2 + 5x - 1$

$+1 \quad +1$

$y + 1 = x^2 + 5x + \frac{25}{4}$

$+\frac{25}{4}$

$b = 5 \quad \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$4 \cdot \frac{1}{4 \cdot 1} + \frac{25}{4}$

LCD = 4

$\frac{4}{4} + \frac{25}{4} = \frac{29}{4}$

$y + \frac{29}{4} = x^2 + 5x + \frac{25}{4}$

$y + \frac{29}{4} = (x + \frac{5}{2})(x + \frac{5}{2})$

$y + \frac{29}{4} = (x + \frac{5}{2})^2$

$-\frac{29}{4} \quad -\frac{29}{4}$

$\boxed{y = (x + 5/2)^2 - 29/4}$

d) $y = x^2 + \frac{1}{4}x - 3$

$+3 \quad +3$

$y + 3 = x^2 + \frac{1}{4}x + \frac{1}{64}$

$\frac{1}{64}$

$b = \frac{1}{4} \quad \frac{1}{4} \cdot \frac{1}{2} = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

$\frac{64 \cdot 3}{64 \cdot 1} + \frac{1}{64} = \frac{192}{64} + \frac{1}{64} = \frac{193}{64}$

LCD = 64

$y + \frac{193}{64} = x^2 + \frac{1}{4}x + \frac{1}{64}$

$y + \frac{193}{64} = (x + \frac{1}{8})(x + \frac{1}{8})$

$y + \frac{193}{64} = (x + \frac{1}{8})^2$

$-\frac{193}{64} \quad -\frac{193}{64}$

$\boxed{y = (x + \frac{1}{8})^2 - \frac{193}{64}}$