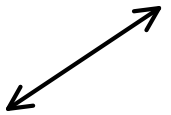
Linear Equations in Two Variables

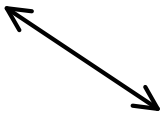
Slope - the ratio of the vertical rise to the horizontal run

$m = \dfrac{\text{rise}}{\text{run}}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$



m = positive



m = negative

$m = 0$

m = undefined / no slope

General Form - use when asked to write the equation of a line in general form

$$Ax + By + C = 0$$

$3x - 4y - 7 = 0$

Is in general form

$-2x - 6y = 8$

$-8 \quad -8$

$\dfrac{-2x}{-1} - \dfrac{6y}{-1} - \dfrac{8}{-1} = \dfrac{0}{-1}$

$2x + 6y + 8 = 0$

Standard Form - use when asked to write the equation of a line in standard form

$$Ax + By = C$$

$2x - 3y = 10$

Is in standard form

$-5x - y = 9$

$\dfrac{-5x}{-1} - \dfrac{1y}{-1} = \dfrac{9}{-1}$

$5x + 1y = -9$

Slope-Intercept Form - use when asked to graph a line

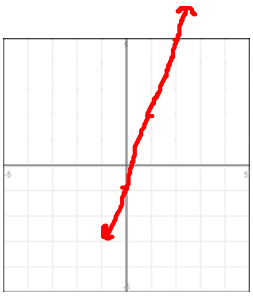
$y = mx + b$ (slope, y-int.)

$y = 3x - 1$

$y = mx + b$

$m = \frac{3}{1}$ ↑ 3, → 1

$b = -1$ $(0, -1)$

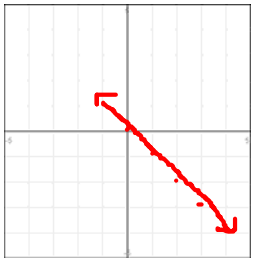


$y = -x$

$y = -1x$

$y = mx + b$

$m = \frac{-1}{1}$ ↓1, →1 $b = 0$ $(0,0)$



Point-Slope Form - use when asked to write the equation of a line

$y - y_1 = m(x - x_1)$ m = slope (x_1, y_1) is a point on the line

Given: $m = \frac{1}{2}$, $(4, -3)$ x_1, y_1

$y - y_1 = m(x - x_1)$

$y - -3 = \frac{1}{2}(x - 4)$

$y + 3 = \frac{1}{2}(x - 4)$

Given: $(-5, 2)$, $(4, -3)$ x_1, y_1 x_2, y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{4 - -5} = \frac{-5}{9}$

$m = \frac{-5}{9}$ $(-5, 2)$ x_1, y_1

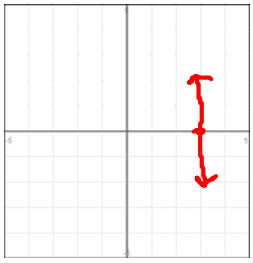
$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{5}{9}(x - -5)$

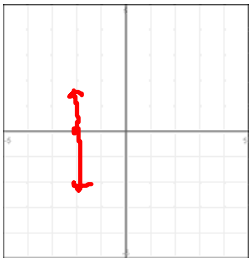
$y - 2 = -\frac{5}{9}(x + 5)$

Vertical Line - an equation of the form $x = c$ m = undefined

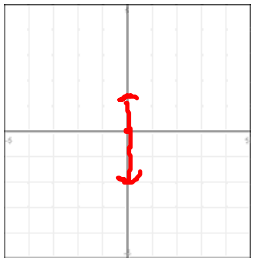
$x = 3$



$x = -2$

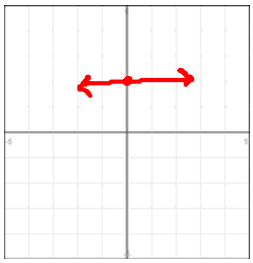


$x = 0$

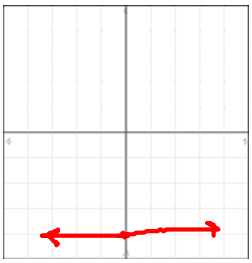


Horizontal Line - an equation of the form $y = c$ m=0

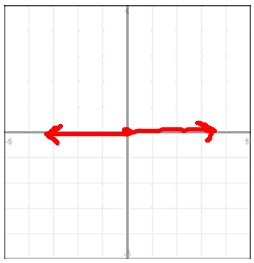
$y = 2$



$y = -4$



$y = 0$



Parallel Lines - have equal slopes symbol: ||

$m = 2$

$m_{\parallel} = 2$

$m = 0$

$m_{\parallel} = 0$

$m = -\frac{4}{3}$

$m_{\parallel} = -\frac{4}{3}$

Perpendicular Lines - have negative, reciprocal slopes symbol: $\perp$

$m = 2$

$m_{\perp} = -\frac{1}{2}$

$m = 0$

horizontal

$m_{\perp}$ = undefined

vertical

$m = -\frac{4}{3}$

$m_{\perp} = \frac{3}{4}$

1. Write each equation in standard form and in slope-intercept form. Identify the slope and the y-intercept.

a) $6y = -2x - 13$

Standard Form

$Ax + By = C$

$6y = -2x - 13$

$+2x \quad +2x$

$2x + 6y = -13$

Slope-Int. Form

$y = mx + b$

$\frac{6y}{6} = \frac{-2x}{6} - \frac{13}{6}$

$y = -\frac{1}{3}x - \frac{13}{6}$

$m = -\frac{1}{3}$ $\quad b = -\frac{13}{6}$

y-int: $\left(0, -\frac{13}{6}\right)$

b) $x = \frac{2}{5}y + 7$

Standard Form

$\frac{5}{5} \cdot \frac{x}{1} = \frac{2y}{5} + \frac{7}{1} \cdot \frac{5}{5}$

LCD = 5

$\frac{5x}{5} = \frac{2y}{5} + \frac{35}{5}$

$\rightarrow 5x = 2y + 35$

$-2y \quad -2y$

$5x - 2y = 35$

Slope-Int. Form

$5x = 2y + 35$

$2y + 35 = 5x$

$-35 \quad -35$

$\frac{2y}{2} = \frac{5x}{2} - \frac{35}{2}$

$y = \frac{5}{2}x - \frac{35}{2}$

$m = \frac{5}{2}$ $\quad b = -\frac{35}{2}$

y-int: $(0, -35/2)$

c) $3x = 7y$

Standard Form

$3x = 7y$

$-7y \quad -7y$

$\boxed{3x - 7y = 0}$

Slope-Int. Form

$3x = 7y$

$\frac{7y}{7} = \frac{3x}{7}$

$\boxed{y = \frac{3}{7}x}$

$\boxed{m = \frac{3}{7}}$ $\quad b = 0$

$\boxed{y\text{-int: } (0,0)}$

2. Write an equation in slope-intercept form for the line that contains the given point and the given slope.

a) $m = -4$, $(-2, -5)$ $\quad (x_1, y_1)$ $\quad y - y_1 = m(x - x_1)$

$y - -5 = -4(x - -2)$

$y + 5 = -4(x + 2)$

$y + 5 = -4x - 8$

$-5 \qquad -5$

$\boxed{y = -4x - 13}$

b) $m = \frac{1}{3}$, $(2, -4)$ $\quad (x_1, y_1)$

$y - -4 = \frac{1}{3}(x - 2)$

$y + 4 = \frac{1}{3}(x - 2)$

$y + 4 = \frac{1}{3}x - \frac{2}{3}$

$-4 \qquad -4$

$\boxed{y = \frac{1}{3}x - \frac{14}{3}}$

$-\frac{2}{3} + \frac{-4 \cdot 3}{1 \cdot 3} = -\frac{2}{3} + \frac{-12}{3} = -\frac{14}{3}$

c) $m = 0$, $(1,7)$

Horizontal Line

$y = c$

$\boxed{y = 7}$

d) $m =$ undefined, $(0,-4)$

Vertical Line

$x = c$

$\boxed{x = 0}$

3. Write an equation in slope-intercept form for the line that contains the given points.

a) $(-7,-3)$, $(6,8)$ $\quad x_1 y_1 \quad x_2 y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - -3}{6 - -7} = \frac{11}{13}$ $\qquad m = \frac{11}{13}$

$y - y_1 = m(x - x_1)$

$y - -3 = \frac{11}{13}(x - -7)$

$y + 3 = \frac{11}{13}(x + 7)$

$y + 3 = \frac{11}{13}x + \frac{77}{13}$

$-3 \qquad -3$

$\frac{77}{13} - \frac{3 \cdot 13}{1 \cdot 13} = \frac{77}{13} - \frac{39}{13} = \frac{38}{13}$

$\boxed{y = \frac{11}{13}x + \frac{38}{13}}$

b) $(-3,4)$, $(-3,7)$ $\quad x_1 y_1 \quad x_2 y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{-3--3} = \frac{3}{0}$

$m =$ undefined

vertical line

$x = c$

$\boxed{x = -3}$

c) $(0,8)$, $(2,8)$ $\quad x_1 y_1 \quad x_2 y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-8}{2-0} = \frac{0}{2} = 0$

$m = 0$

horizontal line

$y = c$

$\boxed{y = 8}$

4. Determine whether the lines L_1 and L_2 are parallel, perpendicular or neither.

a) $L_1:(4,8),\ (-4,2)$
$L_2:(3,-5),\ \left(-1,\frac{1}{3}\right)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$m_{L_1}=\frac{2-8}{-4-4}=\frac{-6}{-8}=\frac{3}{4}$

$m_{L_1}=\boxed{\frac{3}{4}}$

$m_{L_2}=\frac{\frac{1}{3}--5}{-1-3}=\frac{\frac{16}{3}}{-4}=\frac{-4}{3}$

$m_{L_2}=\boxed{\frac{-4}{3}}$

$\frac{1}{3}+\frac{5\cdot3}{1\cdot3}=\frac{1}{3}+\frac{15}{3}=\frac{16}{3}$

$\frac{16}{3}\div-4=\frac{16}{3}\cdot\frac{1}{-4_1}=\frac{-4}{3}$

L_1 is perpendicular to L_2

b) $L_1:(0,-7),\ (2,-3)$
$L_2:(-1,-1),\ (5,11)$

$m_{L_1}=\frac{-3--7}{2-0}=\frac{4}{2}=2$

$m_{L_1}=\boxed{2}$

$m_{L_2}=\frac{11--1}{5--1}=\frac{12}{6}=2$

$m_{L_2}=\boxed{2}$

L_1 is parallel to L_2

c) $L_1:(4,1),\ (-4,-15)$
$L_2:(-12,-5),\ (6,4)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$m_{L_1}=\frac{-15-1}{-4-4}=\frac{-16}{-8}=2$

$m_{L_1}=\boxed{2}$

$m_{L_2}=\frac{4--5}{6--12}=\frac{9}{18}=\frac{1}{2}$

$m_{L_2}=\boxed{\frac{1}{2}}$

Neither

5. Write the slope-intercept form of the equation of the line through the given point that is parallel and perpendicular to the given line.

a) Point: $(-5,1)$

→ Line: $x+y=8$

$y-y_1=m(x-x_1)$

$x+y=8$

$-x \quad -x$

$y=-1x+8$

$m=\frac{-1}{1}$

Parallel x_1, y_1

$m_{\parallel}=-1$ $(-5,1)$

$y-1=-1(x--5)$

$y-1=-1(x+5)$

$y-1=-1x-5$

$+1 \quad +1$

$\boxed{y=-1x-4}$

Perpendicular x_1, y_1

$m_{\perp}=1$ $(-5,1)$

$y-1=1(x--5)$

$y-1=1(x+5)$

$y-1=x+5$

$+1 \quad +1$

$\boxed{y=x+6}$

b) Point: $(4,-3)$

→ Line: $y=-7$

Horizontal Line

$m=0$

Parallel

$m_{\parallel}=0$

$y=c$

$\boxed{y=-3}$

Perpendicular

$m_{\perp}=$ undefined

$x=c$

$\boxed{x=4}$

c) Point: $\left(\frac{1}{2},2\right)$

→ Line: $x=6$

Vertical Line

$m=$ undefined

Parallel

$m_{\parallel}=$ undefined

$x=c$

$\boxed{x=\frac{1}{2}}$

Perpendicular

$m=0$

$y=c$

$\boxed{y=2}$