

Evaluating Functions, Finding the Domain and Difference Quotient

1. Evaluate the function at each value and simplify.

a) $f(x) = 5x - 2$

$$\begin{aligned} f(3) &= 5(3) - 2 \\ &= 15 - 2 \\ &= \boxed{13} \end{aligned}$$

$$\begin{aligned} f(x) &= 5x - 2 \\ f(x+2) &= 5(x+2) - 2 \\ &= 5x + 10 - 2 \\ &= \boxed{5x + 8} \end{aligned}$$

b) $f(x) = x^2 - 2x$

$$\begin{aligned} f\left(\frac{3}{2}\right) &= \left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right) \\ &= \frac{9}{4} - \frac{3 \cdot 4}{4} \\ &\text{LCD} = 4 \\ &= \frac{9}{4} - \frac{12}{4} \\ &= \boxed{-\frac{3}{4}} \end{aligned}$$

$$\begin{aligned} f(x) &= x^2 - 2x \\ f(x+2) &= (x+2)^2 - 2(x+2) \\ &= (x+2)(x+2) - 2(x+2) \\ &= x^2 + 2x + 2x + 4 - 2x - 4 \\ &= \boxed{x^2 + 2x} \end{aligned}$$

c) $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 - 1, & x > 1 \end{cases}$

$$\begin{aligned} f(-2) &= \\ &\stackrel{x^2 + 2}{(-2)^2 + 2} \\ &\stackrel{4+2}{=} \boxed{6} \end{aligned}$$

$$\begin{aligned} f(1) &= \\ &\stackrel{x^2 + 2}{1^2 + 2} \\ &\stackrel{1+2}{=} \boxed{3} \end{aligned}$$

$$\begin{aligned} f(2) &= \\ &\stackrel{2x^2 - 1}{2(2)^2 - 1} \\ &\stackrel{2(4) - 1}{=} \\ &\stackrel{8 - 1}{=} \boxed{7} \end{aligned}$$

Domain - the set of allowable x values

Restrictions

→ Fraction

Denominator $\neq 0$

$$\frac{1}{x+2}$$

$$x+2 \neq 0$$

$$-2 -2$$

$$x \neq -2$$

→ Even Root

Radicand ≥ 0

$$\sqrt[2]{x+2}$$

$$x+2 \geq 0$$

$$-2 -2$$

$$x \geq -2$$

$$\frac{1}{\sqrt{x+2}}$$

$$x+2 \geq 0$$

$$-2 -2$$

$$x > -2$$

2. Find the domain of each function.

a) $f(x) = 2x^3 + 1$

$$\boxed{\mathbb{R}}$$

b) $f(x) = \frac{x}{2x+1}$

$$2x+1 \neq 0$$

$$-1 -1$$

$$\frac{2}{2}x \neq \frac{-1}{2}$$

$$\boxed{x \neq -\frac{1}{2}}$$

c) $f(x) = \frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)}$

$$x+1 \neq 0$$

$$-1 -1$$

$$x-1 \neq 0$$

$$+1 +1$$

$$\boxed{x \neq -1 \quad x \neq 1}$$

d) $f(x) = \sqrt{x+1}$

$$x+1 \geq 0$$

$$-1 -1$$

$$\boxed{x \geq -1}$$

$$e) f(x) = \sqrt[3]{x+1}$$

$\boxed{\mathbb{R}}$

$$f) f(x) = \frac{\sqrt{x+2}}{x-5}$$

$$\begin{array}{l} x+2 \geq 0 \\ -2 -2 \\ x \geq -2 \end{array} \quad \begin{array}{l} x-5 \neq 0 \\ +5 +5 \\ x \neq 5 \end{array}$$



$$[-2, 5) \cup (5, \infty)$$

$$[-2 \leq x < 5 \cup x > 5]$$

$$h) f(x) = \frac{x+1}{\sqrt{2x-10}}$$

$$g) f(x) = \sqrt{x^2 + 1}$$

$$\begin{array}{l} x^2 + 1 \geq 0 \\ x^2 + 1 = 0 \\ -1 -1 \\ \sqrt{x^2} = \sqrt{-1} \end{array}$$

$\boxed{\mathbb{R}}$

$$\begin{array}{l} 2x - 10 > 0 \\ +10 +10 \end{array}$$

$$\frac{2x}{2} > \frac{10}{2}$$

$$\boxed{x > 5}$$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

3. Find the difference quotient and simplify your answer.

a) $f(x) = 2x - 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h) - 1 - (2x - 1)}{h}$$

$$\frac{2x + 2h - 1 - 2x + 1}{h} = \frac{2h}{h} = 2$$

b) $f(x) = x^2 - 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3 - (x^2 - 3)}{h}$$

$$\frac{(x+h)(x+h) - 3 - (x^2 - 3)}{h} = \frac{x^2 + xh + xh + h^2 - 3 - x^2 + 3}{h}$$

$$\frac{2xh + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h$$

c) $f(x) = \frac{1}{x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = \frac{-h}{x(x+h)h}$$

$$= \frac{-1}{x(x+h)}$$