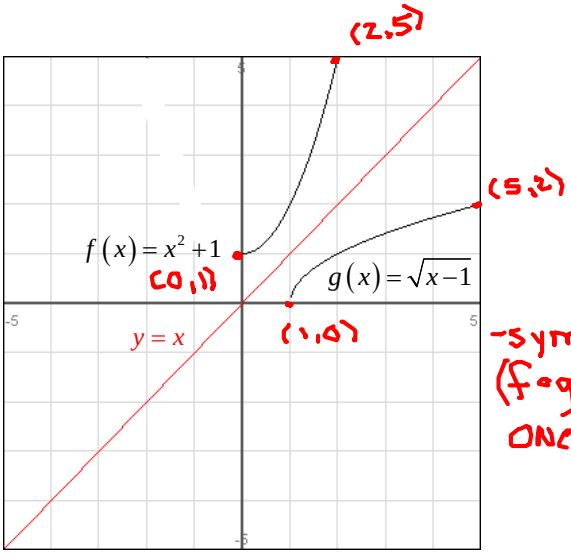
Inverse Functions

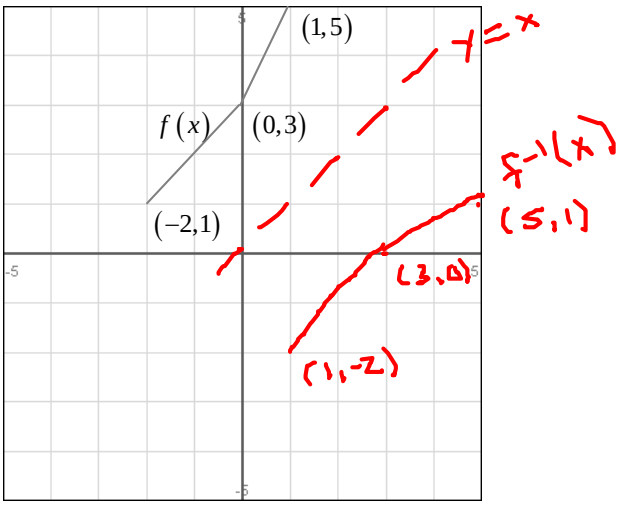


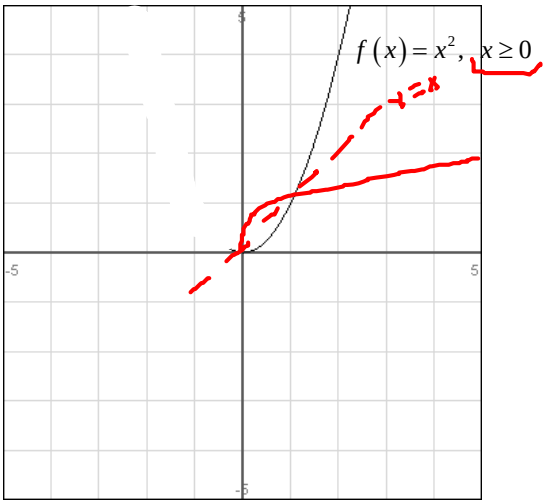
$f(x)=x^2+1$	$g(x)=\sqrt{x-1}$
$D: x \geq 0$	$D: y \geq 1$
$R: y \geq 1$	$R: x \geq 0$
y-intercept: $(0,1)$	x-intercept: $(1,0)$

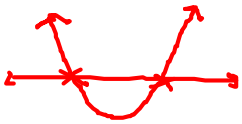
-symmetrical about $y=x$

$(f \circ g)(x) = (g \circ f)(x) = x$

ONE-to-ONE







Steps to find the inverse of a function:

1. Substitute y in for $f(x)$.
2. Interchange x and y.
3. Solve for y.
4. Substitute $f^{-1}(x)$ in for y.

1. Find the inverse function of $f(x)$.

a) $f(x) = 2x - 5$

$y = 2x - 5$

$x = 2y - 5$

$+5 \qquad +5$

$\frac{x+5}{2} = \frac{2y}{2}$

$y = \frac{x+5}{2}$

$\boxed{f^{-1}(x) = \frac{x+5}{2}}$

b) $f(x) = \sqrt[3]{1-3x}$

$y = \sqrt[3]{1-3x}$

$(x)^3 = (\sqrt[3]{1-3y})^3$

$x^3 = 1 - 3y$

$-1 \qquad -1$

$\frac{x^3-1}{-3} = \frac{-3y}{-3}$

$y = \frac{x^3-1}{-3}$

$\boxed{f^{-1}(x) = \frac{x^3-1}{-3}}$

c) $f(x) = x^3 + 2$

$y = x^3 + 2$

$x = y^3 + 2$

$-2 \qquad -2$

$\sqrt[3]{x-2} = \sqrt[3]{y^3}$

$y = \sqrt[3]{x-2}$

$\boxed{f^{-1}(x) = \sqrt[3]{x-2}}$

d) $f(x) = \frac{x+3}{x-2}$

$y = \frac{x+3}{x-2}$

$x = \frac{y+3}{y-2}$

$y+3 = x(y-2)$

$y+3 = xy - 2x$

$-xy - 3 \qquad -xy \quad -3$

$y - xy = -2x - 3$

$\frac{y(1-x)}{1-x} = \frac{-2x-3}{1-x}$

$y = \frac{-2x-3}{1-x}$

$\boxed{f^{-1}(x) = \frac{-2x-3}{1-x}}$

e) $f(x)=\frac{2x+1}{x-1}$

$$y=\frac{2x+1}{x-1}$$

$$\frac{x}{1}=\frac{2y+1}{y-1}$$

$$2y+1=x(y-1)$$

$$2y+1=xy-x$$

$$-xy-1 \quad -xy-1$$

$$2y-xy=-x-1$$

$$\frac{y(2-x)}{2-x}=\frac{-x-1}{2-x}$$

$$y=\frac{-x-1}{2-x}$$

$$\boxed{f^{-1}(x)=\frac{-x-1}{2-x}}$$

2. Show that the functions f and g are inverse functions algebraically and graphically.

a) $f(x) = 2 - 4x, \quad g(x) = \dfrac{2-x}{4}$

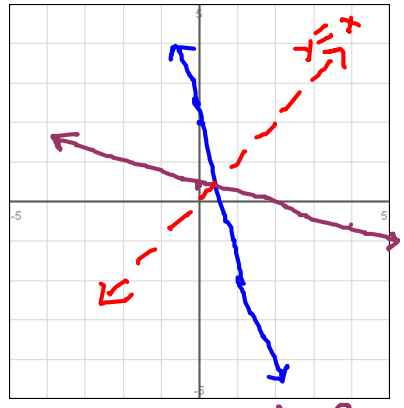
$(f \circ g)(x) = f(g(x)) = 2 - 4\left(\dfrac{2-x}{4}\right)$

$= 2 - \dfrac{\cancel{4}(2-x)}{\cancel{4}} = \cancel{2} - \cancel{2} + x = x$

$(g \circ f)(x) = g(f(x)) = \dfrac{2-(2-4x)}{4}$

$= \dfrac{\cancel{2} - \cancel{2} + 4x}{4} = \dfrac{\cancel{4}x}{\cancel{4}} = x$

$(f \circ g)(x) = (g \circ f)(x) = x$ ✓



$f(x) = -4x + 2$

$m = -4$

$b(0,2)$

$g(x) = \frac{2}{4} - \frac{x}{4}$

$g(x) = -\frac{1}{4}x + \frac{1}{2}$

$m = -\frac{1}{4}$ $b(0, 1/2)$

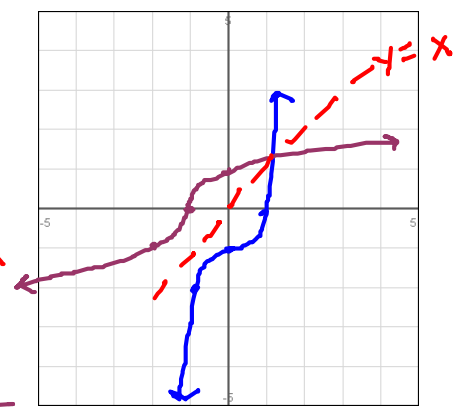
b) $f(x) = x^3 - 1, \quad g(x) = \sqrt[3]{x+1}$

$(f \circ g)(x) = (\sqrt[3]{x+1})^3 - 1 = x + \cancel{1} - \cancel{1} = x$

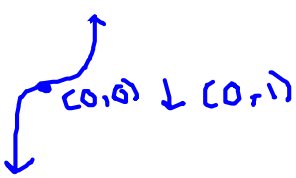
$(g \circ f)(x) = \sqrt[3]{(x^3 - 1) + 1}$

$= \sqrt[3]{x^3 \cancel{-1} \cancel{+1}} = \sqrt[3]{x^3} = x$

$(f \circ g)(x) = (g \circ f)(x) = x$ ✓

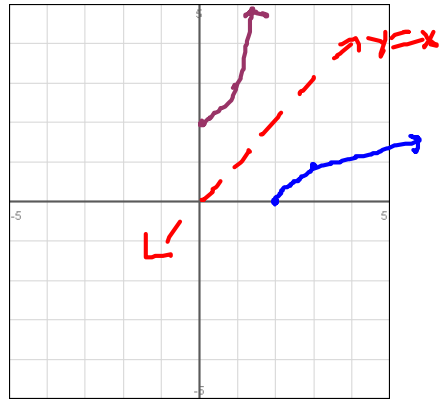


$f(x) = x^3 - 1$



$g(x) = \sqrt[3]{x+1}$

c) $f(x)=\sqrt{x-2},\ \ g(x)=x^2+2\ \underbrace{(x\geq 0)}$



$(f\circ g)(x)=\sqrt{(x^2+2)-2}=\sqrt{x^2+\cancel{2}-\cancel{2}}$

$=\sqrt{x^2}=x$

$(g\circ f)(x)=(\sqrt{x-2})^2+2=x-\cancel{2}+\cancel{2}$

$=x$

$(f\circ g)(x)=(g\circ f)(x)=x$ ✓

$g(x)=x^2+2,\ x\geq 0$

$f(x)=\sqrt{x-2}$

$(0,0)\rightarrow(2,0)$

$(0,0)\uparrow(0,2)$