Quadratic Functions

Let a, b and c be real numbers with $a \neq 0$.
$f(x) = ax^2 + bx + c$ is called a quadratic function.
$f(x) = a(x-h)^2 + k$ is the standard form of a quadratic function.

$\underline{f(x) = ax^2 + bx + c}$

vertex: $(x, y) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$x-$intercept: set $y = 0$

$y-$intercept: set $x = 0$

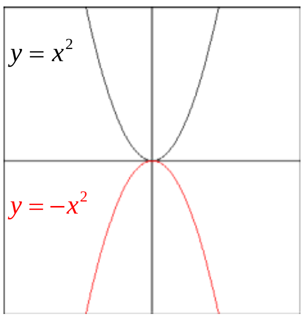
$\underline{f(x) = a(x-h)^2 + k}$

vertex: (h, k)

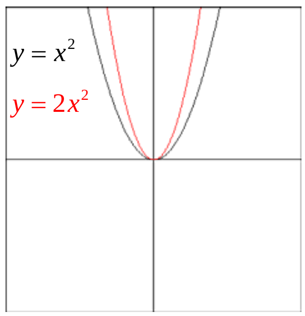
$x-$intercept: set $y = 0$

$y-$intercept: set $x = 0$

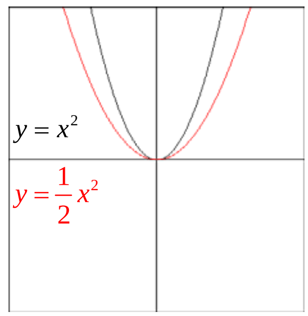
$\underline{a < 0}$



$\underline{a > 1}$



$\underline{0 < a < 1}$



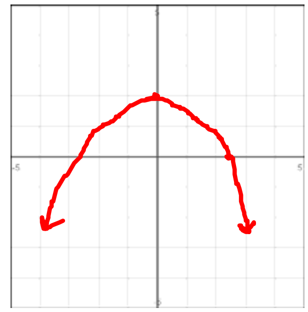
1. Sketch the graph of each quadratic function. Identify the vertex and the x and y intercepts.

a) $f(x) = 2 - \frac{1}{4}x^2$

$f(x) = -\frac{1}{4}x^2 + 2$

vertex $\boxed{(0,2)}$

$a = -\frac{1}{4}$



X-INT set $y=0$

$0 = -\frac{1}{4}x^2 + 2$

$-2 \qquad -2$

$-4 \cdot -2 = -4 \cdot -\frac{1}{4}x^2$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm 2\sqrt{2}$

$x = \pm 2.8$

$\boxed{(2.8, 0) \quad (-2.8, 0)}$

Y-INT set $x=0$

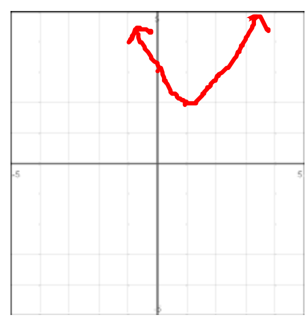
$y = -\frac{1}{4}(0)^2 + 2$

$y = 2$

$\boxed{(0,2)}$

b) $f(x) = (x-1)^2 + 2$

vertex $\boxed{(1,2)}$ $a = 1$



X-INT

$0 = (x-1)^2 + 2$

$-2 \qquad -2$

$\sqrt{-2} = \sqrt{(x-1)^2}$

$\boxed{\text{NO X-INT}}$

Y-INT

$y = (0-1)^2 + 2$

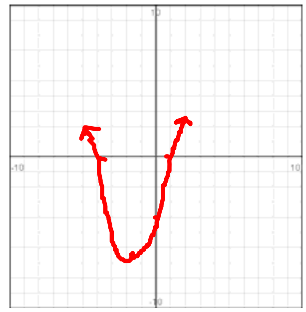
$y = (-1)^2 + 2$

$y = 1 + 2$

$y = 3$

$\boxed{(0,3)}$

c) $f(x)=x^2+3x-4$ $a=1$ $b=3$



vertex: $x=\frac{-b}{2a}=\frac{-3}{2(1)}=\frac{-3}{2}$

$\boxed{(-1.5,-6.25)}$

$y=\left(\frac{-3}{2}\right)^2+3\left(\frac{-3}{2}\right)-4=\frac{9}{4}-\frac{9\cdot 2}{2\cdot 2}-\frac{4\cdot 4}{1\cdot 4}$

LCD=4

$=\frac{9}{4}-\frac{18}{4}-\frac{16}{4}=\frac{-25}{4}=-6.25$

x-int

$0=x^2+3x-4$

$0=(x+4)(x-1)$

$x+4=0$ $x-1=0$

$x=-4$ $x=1$

$\boxed{(-4,0)\quad(1,0)}$

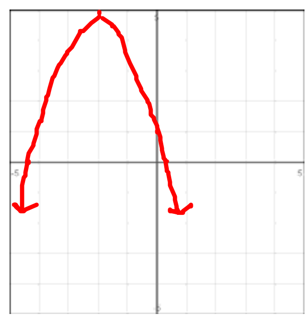
y-int

$y=0^2+3(0)-4$

$y=-4$

$\boxed{(0,-4)}$

d) $f(x)=-x^2-4x+1$ $a=-1$ $b=-4$ $c=1$



vertex

$\boxed{(-2,5)}$

$x=\frac{-b}{2a}=\frac{-(-4)}{2(-1)}=\frac{4}{-2}=-2$

$y=-(-2)^2-4(-2)+1$

$=-4+8+1$

$=5$

x-int

$0=-x^2-4x+1$

$-1\quad -1\quad -1\quad -1$

$0=x^2+4x-1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(-1)(1)}}{2(-1)}$

$x=\frac{4\pm\sqrt{20}}{-2}$

$x=\frac{\cancel{4}\pm\cancel{2}\sqrt{5}}{\cancel{-2}}$ -2

$x=-2\pm\sqrt{5}$

$x=.24, -4.24$

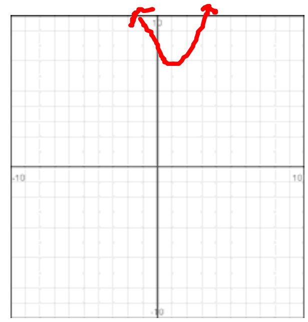
$\boxed{(.24,0)(-4.24,0)}$

y-int

$y=-(0)^2-4(0)+1$

$y=1$ $\boxed{(0,1)}$

e) $f(x) = x^2 - 2x + 8$ $a=1$ $b=-2$ $c=8$



vertex

$\boxed{(1,7)}$

$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$y = (1)^2 - 2(1) + 8 = 1 - 2 + 8 = 7$

x-int

$0 = x^2 - 2x + 8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-28}}{2}$

$\boxed{\text{NO x-int.}}$

y-int

$y = 0^2 - 2(0) + 8$

$y = 8$

$\boxed{(0,8)}$

2. Rewrite the quadratic function in standard form using the method of completing the square and then identify the vertex. $f(x) = ax^2 + bx + c \longrightarrow f(x) = a(x-h)^2 + k$

a) $f(x) = -(x^2 - 2x + 10)$

$f(x) = (-x^2 + 2x) - 10$

$f(x) = -1(x^2 - 2x + 1) - 10 + 1$

$\frac{2}{2} = 1^2 = 1$

$\boxed{f(x) = -1(x-1)^2 - 9}$

vertex: $\boxed{(1,-9)}$

b) $f(x) = (4x^2 + 24x) + 5$

$f(x) = 4(x^2 + 6x + 9) + 5 - 36$

$\frac{6}{2} = 3^2 = 9$

$\boxed{f(x) = 4(x+3)^2 - 31}$ vertex: $\boxed{(-3, -31)}$

c) $f(x) = (x^2 + 3x) + 5$

$f(x) = 1(x^2 + 3x + \frac{9}{4}) + \frac{4 \cdot 5}{4 \cdot 1} - \frac{9}{4} = \frac{20}{4} - \frac{9}{4} = \frac{11}{4}$

$\frac{3}{2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$\boxed{f(x) = (x + \frac{3}{2})^2 + \frac{11}{4}}$ $\boxed{\text{vertex: } (-3/2, 11/4)}$

d) $f(x) = \frac{1}{2}(x^2 + x + 8)$

$f(x) = (\frac{1}{2}x^2 + \frac{1}{2}x) + 4$

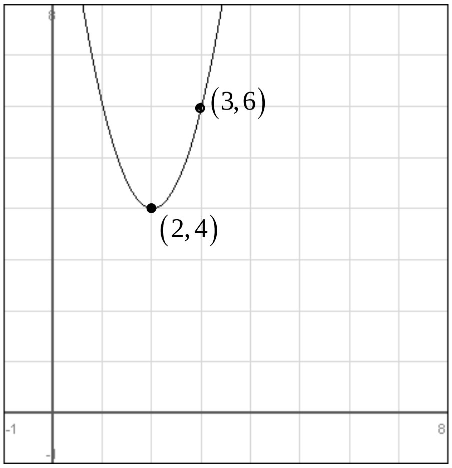
$f(x) = \frac{1}{2}(x^2 + x + \frac{1}{4}) + \frac{8 \cdot 4}{8 \cdot 1} - \frac{1}{8} = \frac{32}{8} - \frac{1}{8} = \frac{31}{8}$

$\frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ LCD = 8

$\boxed{f(x) = \frac{1}{2}(x + \frac{1}{2})^2 + \frac{31}{8}}$ $\boxed{\text{vertex: } (-1/2, 31/8)}$

3. Find an equation for the parabola.

a)



$y = a(x-h)^2 + K$

vertex $\overset{h}{(2},\overset{K}{4)}$

$\overset{x}{(3},\overset{y}{6)}$

$6 = a(3-2)^2 + 4$

$6 = a(1)^2 + 4$

$6 = a + 4$

$-4 \qquad -4$

$a = 2$

$\boxed{y = 2(x-2)^2 + 4}$

b) Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right)$, Point: $(-2, 1)$

h K   x y

$y = a(x-h)^2 + K \leftarrow$

$1 = a\left(-2 - -\frac{1}{4}\right)^2 + \frac{3}{2}$

$1 = a\left(\frac{4 \cdot -2}{4 \cdot 1} + \frac{1}{4}\right)^2 + \frac{3}{2}$

LCD = 4

$-\frac{8}{4} + \frac{1}{4} = \left(\frac{-7}{4}\right)^2 = \frac{49}{16}$

$1 = a\left(\frac{49}{16}\right) + \frac{3}{2}$

$-\frac{3}{2} \qquad -\frac{3}{2}$

$\frac{\cancel{16}^{8}}{49} \cdot -\frac{1}{\cancel{2}_{1}} = a\left(\frac{49}{\cancel{16}}\right) \cdot \frac{16}{49}$

$a = -\frac{8}{49}$

$\boxed{y = \frac{-8}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}}$