Factoring

Greatest Common Factor (GCF)

Directions: Factor.

1. $24y - 48$

$\frac{24y}{24} \quad \frac{48}{24}$

GCF = 24

$\boxed{24(y-2)}$

2. $3x^5 - 15x^2$

$\frac{3x^5}{3x^2} \quad \frac{15x^2}{3x^2}$

GCF = $3x^2$

$\boxed{3x^2(x^3-5)}$

3. $16a^2b^1 + 32a^2b^2$

$\frac{16a^2b^1}{16a^2b^1} \quad \frac{32a^2b^2}{16a^2b^1}$

GCF = $16a^2b^1$

$\boxed{16a^2b(1+2b)}$

4. $8x^2y - 12xy^2 + 5xy^1$

$\frac{8x^2y}{x^1y^1} \quad \frac{12xy^2}{x^1y^1} \quad \frac{5xy^1}{x^1y^1}$

GCF = x^1y^1

$\boxed{xy(8x-12y+5)}$

5. $x(x+2) + 3(x+2)$

$\frac{x(x+2)}{x+2} \quad \frac{3(x+2)}{x+2}$

GCF = $x+2$

$\boxed{(x+2)(x+3)}$

6. $21x^4 - 7x^1$

$\frac{21x^4}{7x^1} \quad \frac{7x^1}{7x^1}$

GCF = $7x^1$

$\boxed{7x(3x^3-1)}$

Difference of Two Squares

Use when:

- Two Terms
- Both terms are perfect squares
- Subtraction symbol in between the terms

$$a^2 - b^2 = (a+b)(a-b)$$

a a b b

Square Root	Perfect Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

Directions: Factor.

1. $x^2 - 81$

x x 9 9

$\boxed{(x+9)(x-9)}$

2. $9x^2 - 25y^2$

3x 3x 5y 5y

$\boxed{(3x+5y)(3x-5y)}$

3. $8 - 8a^4$ GCF = 8

$\frac{8}{8}$ $\frac{8}{8}$

$8(1 - a^4)$

1 1 a^2 a^2

$8(1+a^2)(1-a^2)$

1 1 a a

$\boxed{8(1+a^2)(1+a)(1-a)}$

4. $2x^4 - 50$ GCF = 2

$\frac{2}{2}$ $\frac{2}{2}$

$2(x^4 - 25)$

x^2 x^2 5 5

$\boxed{2(x^2+5)(x^2-5)}$

5. $x^5y^2 - 4x^3y^4$ GCF = x^3y^2

$\frac{x^5y^2}{x^3y^2}$ $\frac{4x^3y^4}{x^3y^2}$

$x^3y^2(x^2 - 4y^2)$

x x 2y 2y

$\boxed{x^3y^2(x+2y)(x-2y)}$

Sum and Difference of Two Cubes

Use when:

- Two Terms
- Both terms are perfect cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

a a a b b b

Cube Root	Perfect Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1,000

Directions: Factor.

1. $x^3 + 27$

x x x 3 3 3

A= x B=3

$(A+B)(A^2-AB+B^2)$

$\boxed{(x+3)(x^2-3x+9)}$

2. $2x^3 - 16$ GCF=2

$\frac{2x^3}{2} - \frac{16}{2}$

$2(x^3-8)$

x x x 2 2 2

A=x B=2

$(A-B)(A^2+AB+B^2)$

$\boxed{2(x-2)(x^2+2x+4)}$

3. $125 - y^3$

5 5 5 y y y

A=5 B=y

$(A-B)(A^2+AB+B^2)$

$\boxed{(5-y)(25+5y+y^2)}$

4. $64x^3 + 729y^6$

4x 4x 4x $9y^2$ $9y^2$ $9y^2$

A=4x B=$9y^2$

$(A+B)(A^2-AB+B^2)$

$\boxed{(4x+9y^2)(16x^2-36xy^2+81y^4)}$

5. $16a^4 + 250\cancel{a}$ GCF=2a

$\frac{16a^4}{2a} + \frac{250\cancel{a}}{2\cancel{a}}$

$2a(8a^3+125)$

2a 2a 2a 5 5 5

A=2a B=5

$(A+B)(A^2-AB+B^2)$

$\boxed{2a(2a+5)(4a^2-10a+25)}$

Factor By Grouping

Use when:

- Four Terms

Directions: Factor.

1. $x^2 - 4x + 6x - 24$

$x \quad x \quad 6 \quad 6$

GCF=x GCF=6

$\frac{x(x-4)}{x-4} + \frac{6(x-4)}{x-4}$

GCF= x-4

$\boxed{(x-4)(x+6)}$

2. $x^3 - 3x^2 + 2x^2 - 6$

$x \quad x \quad 2 \quad 2$

GCF=x GCF=2

$\frac{x(x^2-3)}{x^2-3} + \frac{2(x^2-3)}{x^2-3}$

GCF= x^2-3

$\boxed{(x^2-3)(x+2)}$

3. $3y - 9 - xy + 3x$

$3 \quad 3 \quad -x \quad -x$

GCF=3 GCF=-x

$\frac{3(y-3)}{y-3} - \frac{x(y-3)}{y-3}$

GCF= (y-3)

$\boxed{(y-3)(3-x)}$

4. $5x^2 + 20x - x - 4$

$5x \quad 5x \quad -1 \quad -1$

GCF=5x GCF=-1

$\frac{5x(x+4)}{x+4} - \frac{1(x+4)}{x+4}$

GCF= x+4

$\boxed{(x+4)(5x-1)}$

5. $3x^3 + x^2 - 12x - 4$

$x^2 \quad x^2 \quad -4 \quad -4$

GCF=x^2 GCF=-4

$\frac{x^2(3x+1)}{3x+1} - \frac{4(3x+1)}{3x+1}$

GCF= (3x+1)

$(3x+1)(x^2-4) = \boxed{(3x+1)(x+2)(x-2)}$

x x 2 2

Factoring Trinomials

Type 1: $a = 1$

$$ax^2 + bx + c$$

1st term → ax^2; 1st sign: $+$; middle term: bx; 2nd sign: $+$; last term: c

Step 1: Look for GCF.

Step 2: Find the two numbers that multiply to the last term and add or subtract (depending on the second sign) to the middle term.

Step 3: If the second sign is positive, then both signs inside the parentheses get the first sign.
If the second sign is negative, then the larger number inside the parentheses gets the first sign.

Directions: Factor.

1. $y^2 - 8y + 12$

$1 \cdot 12$
$\boxed{2 \cdot 6}$
$3 \cdot 4$

$\boxed{(y - 2)(y - 6)}$

2. $x^2 + 14x + 49$

$1 \cdot 49$
$\boxed{7 \cdot 7}$

$\boxed{(x + 7)(x + 7) \text{ OR } (x + 7)^2}$

3. $x^2 - 13x - 48$

$1 \cdot 48$
$2 \cdot 24$
$\boxed{3 \cdot 16}$
$4 \cdot 12$
$6 \cdot 8$

$\boxed{(x + 3)(x - 16)}$

4. $-m^2 - 9m - 18$ GCF $= -1$

$\frac{-m^2}{-1} \quad \frac{-9m}{-1} \quad \frac{-18}{-1}$

$-1(m^2 + 9m + 18)$

$1 \cdot 18$
$2 \cdot 9$
$\boxed{3 \cdot 6}$

$\boxed{-1(m + 3)(m + 6)}$

5. $3x^2 - 27x + 42$ GCF $= 3$

$\frac{3x^2}{3} \quad \frac{-27x}{3} \quad \frac{42}{3}$

$3(x^2 - 9x + 14)$

$1 \cdot 14$
$\boxed{2 \cdot 7}$

$\boxed{3(x - 2)(x - 7)}$

Type 2: $a \neq 1$ Trial and error

Step 1: Look for GCF.

Step 2: List the factors on the first and last terms.

Step 3: Use the numbers whose outer and inner products add or subtract (depending on the second sign) to the middle term.

Step 4: If the second sign is positive, then both signs inside the parentheses get the first sign. If the second sign is negative, then the larger product inside the parentheses gets the first sign.

Directions: Factor.

1. $2x^2 + 9x + 4$

$2 \cdot 1$ $1 \cdot 4$ $2 \cdot 2$

$(2x + 1)(x + 4)$

$1x$

$9x$

$\boxed{(2x+1)(x+4)}$

2. $3x^2 - 2x - 8$

$1 \cdot 3$ $1 \cdot 8$ $2 \cdot 4$

$(3x + 4)(x - 2)$

$4x$

$6x$

$\boxed{(3x+4)(x-2)}$

3. $7x^2 - 8x + 1$

$1 \cdot 7$ $1 \cdot 1$

$(7x - 1)(x - 1)$

$1x$

$7x$

$\boxed{(7x-1)(x-1)}$

4. $15x^2 - 1xy - 6y^2$

$1 \cdot 15$ $1 \cdot 6$

$\rightarrow 3 \cdot 5$ $\rightarrow 2 \cdot 3$

X $(3x \quad 3y)(5x \quad 2y)$

$15xy$

$6xy$

$(3x - 2y)(5x + 3y)$

$10xy$

$9xy$

$\boxed{(3x-2y)(5x+3y)}$