# Solving Quadratic Equations by Completing the Square

Quadratic Equation - An equation of the form $ax^2+bx+c=0$, where $a$, $b$ and $c$ are real numbers and $a \neq 0$.

Step 1: Isolate the square and linear terms.
Step 2: Divide the coefficient of the linear term by two and then square it. Add the number to both sides of the equation.
Step 3: Write in factored form and the use the square root method to solve for the variable.

Directions: Solve each quadratic equation by completing the square.

1. $x^2+4x+3=0$

$-3 \quad -3$

$x^2+4x+4=-3+4$

$\frac{4}{2}=2^2=4$

$x^2+4x+4=1$

$(x+2)(x+2)=1$

$\sqrt{(x+2)^2}=\sqrt{1}$

$x+2=\pm 1$

$x+2=1 \quad x+2=-1$

$-2 \quad -2 \quad -2 \quad -2$

$x=-1 \quad x=-3$

2. $x^2-8x+12=0$

$-12 \quad -12$

$x^2-8x+16=-12+16$

$\frac{8}{2}=(4)^2=16$

$x^2-8x+16=4$

$(x-4)(x-4)=4$

$\sqrt{(x-4)^2}=\sqrt{4}$

$x-4=\pm 2$

$x-4=2 \quad x-4=-2$

$+4 \quad +4 \quad +4 \quad +4$

$x=6 \quad x=2$

3. $x^2 + 3x = 4$

$x^2 + 3x + \frac{9}{4} = 4 + \frac{9}{4}$

$\frac{3}{2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$\frac{4 \cdot 4}{4 \cdot 1} + \frac{9}{4} = \frac{16}{4} + \frac{9}{4} = \frac{25}{4}$

$x^2 + 3x + \frac{9}{4} = \frac{25}{4}$

$\left(x + \frac{3}{2}\right)\left(x + \frac{3}{2}\right) = \frac{25}{4}$

$\sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{25}{4}}$

$x + \frac{3}{2} = \pm \frac{5}{2}$

$x + \frac{3}{2} = \frac{5}{2}$ $\quad$ $x + \frac{3}{2} = -\frac{5}{2}$

$-\frac{3}{2} \quad -\frac{3}{2}$ $\quad$ $-\frac{3}{2} \quad -\frac{3}{2}$

$\boxed{x = 1}$ $\quad$ $\boxed{x = -4}$

4. $3x^2 + 12x - 18 = 0$

$+18 \quad +18$

$\frac{3x^2}{3} + \frac{12x}{3} = \frac{18}{3}$

$x^2 + 4x + 4 = 6 + 4$

$\frac{4}{2} = 2^2 = 4$

$x^2 + 4x + 4 = 10$

$(x + 2)(x + 2) = 10$

$\sqrt{(x + 2)^2} = \sqrt{10}$

$x + 2 = \pm\sqrt{10}$

$x + 2 = \sqrt{10}$ $\quad$ $x + 2 = -\sqrt{10}$

$-2 \quad -2$ $\quad$ $-2 \quad -2$

$\boxed{x = -2 + \sqrt{10}}$ $\quad$ $\boxed{x = -2 - \sqrt{10}}$

5. $5x^2 - 5x + 12 = 8$

$-12 \quad -12$

$\frac{5x^2}{5} - \frac{5x}{5} = \frac{-4}{5}$

$x^2 - 1x + \frac{1}{4} = -\frac{4}{5} + \frac{1}{4}$

$\frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\frac{-4 \cdot 4}{5 \cdot 4} + \frac{1 \cdot 5}{4 \cdot 5} = \frac{-16}{20} + \frac{5}{20} = \frac{-11}{20}$

$x^2 - 1x + \frac{1}{4} = -\frac{11}{20}$

$\left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) = -\frac{11}{20}$

$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{-\frac{11}{20}}$

$x - \frac{1}{2} = \pm \frac{\sqrt{-11}}{\sqrt{20}}$ $\qquad \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

$x - \frac{1}{2} = \pm \frac{i\sqrt{11}}{2\sqrt{5}}$

$+\frac{1}{2} \qquad +\frac{1}{2}$

$x = \frac{1}{2} \pm \frac{i\sqrt{11}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$x = \frac{1 \cdot 5}{2 \cdot 5} \pm \frac{i\sqrt{55}}{10}$ $\qquad LCD = 10$

$x = \frac{5}{10} \pm \frac{i\sqrt{55}}{10}$

$$\boxed{x = \frac{5 \pm i\sqrt{55}}{10}}$$

6. $9x^2 - 12x = -10$

$\frac{9}{9} \quad \frac{9}{9} \quad \frac{9}{9}$

$x^2 - \frac{4}{3}x + \frac{4}{9} = -\frac{10}{9} + \frac{4}{9}$

$\frac{4}{3} \cdot \frac{1}{2} = \frac{4}{6} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$x^2 - \frac{4}{3}x + \frac{4}{9} = -\frac{6}{9}$

$\left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = -\frac{2}{3}$

$\sqrt{\left(x - \frac{2}{3}\right)^2} = \sqrt{-\frac{2}{3}}$

$x - \frac{2}{3} = \pm \sqrt{\frac{-2}{3}}$

$x - \frac{2}{3} = \pm \frac{i\sqrt{2}}{\sqrt{3}}$

$+\frac{2}{3} \qquad +\frac{2}{3}$

$x = \frac{2}{3} \pm \frac{i\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{2}{3} \pm \frac{i\sqrt{6}}{3}$

$$\boxed{x = \frac{2 \pm i\sqrt{6}}{3}}$$

7. $-x^2+7x-9=2$

$+9 \quad +9$

$\frac{-x^2}{-1}+\frac{7x}{-1}=\frac{11}{-1}$

$x^2-7x+\frac{49}{4}=-11+\frac{49}{4}$

$\frac{7}{2}\div\left(\frac{7}{2}\right)^2=\frac{49}{4}$

$\frac{-11\cdot 4}{4\cdot 1}+\frac{49}{4}=-\frac{44}{4}+\frac{49}{4}=\frac{5}{4}$

$x^2-7x+\frac{49}{4}=\frac{5}{4}$

$\left(x-\frac{7}{2}\right)\left(x-\frac{7}{2}\right)=\frac{5}{4}$

$\sqrt{\left(x-\frac{7}{2}\right)^2}=\sqrt{\frac{5}{4}}$

$x-\frac{7}{2}=\pm\frac{\sqrt{5}}{2}$

$+\frac{7}{2} \quad +\frac{7}{2}$

$x=\frac{7}{2}\pm\frac{\sqrt{5}}{2}$

$\boxed{x=\frac{7\pm\sqrt{5}}{2}}$