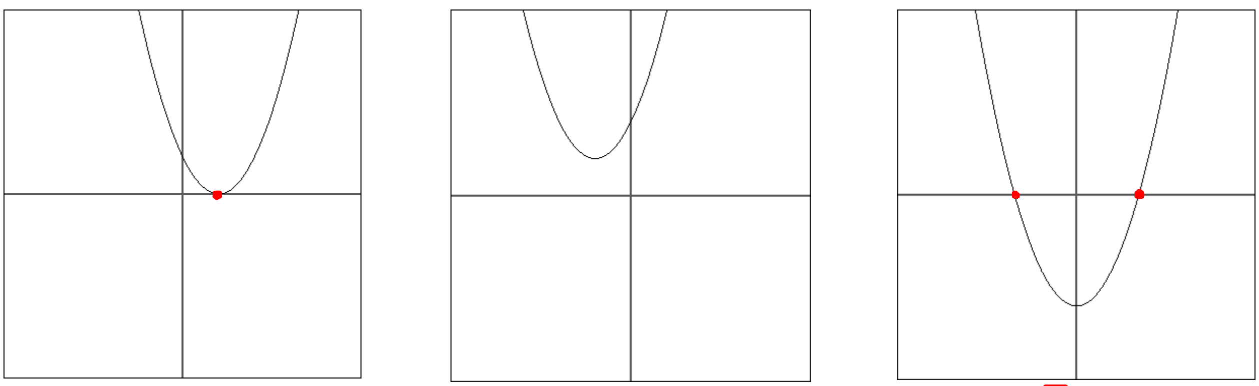
The Discriminant

Quadratic Equation - An equation of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

Discriminant - Determines the number and type of roots of a quadratic equation when a, b and c are rational numbers.

Quadratic Formula	Discriminant
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$D = b^2 - 4ac$

One Root | Zero Roots | Two Roots



1 | 2 | 3

Value of the Discriminant	Number and Type of Roots	
Negative	No Solution - Two Imaginary Roots	2
Zero	One Solution - One Real, Rational Root	1
Positive Perfect Square	Two Solutions - Two Real, Rational Roots	3
Positive Non-Perfect Square	Two Solutions - Two Real, Irrational Roots	3

Directions: Use the discriminant to determine the number and type of solutions to the quadratic equation.

Nature of the roots

1. $2x^2 + 6x + 3 = 0$

$D = b^2 - 4ac$

$a = 2 \quad b = 6 \quad c = 3$

$D = (6)^2 - 4(2)(3)$

$= 36 - 24$

$= 12$ positive, not a perfect square

$\boxed{\text{2 solutions, real and irrational}}$

2. $x^2 - 4x = -5$

$+5 \quad +5$

$x^2 - 4x + 5 = 0$

$a = 1 \quad b = -4 \quad c = 5$

$D = (-4)^2 - 4(1)(5)$

$= 16 - 20$

$= -4$ Negative

$\boxed{\text{No solution, 2 imaginary roots}}$

3. $x^2 - 2x - 3 = 0$

$D = b^2 - 4ac$

$a = 1 \quad b = -2 \quad c = -3$

$D = (-2)^2 - 4(1)(-3)$

$= 4 + 12$

$= 16$ positive, perfect square

$\boxed{\text{2 solutions, real and rational}}$

4. $x^2 - 6x + 9 = 0$

$a = 1 \quad b = -6 \quad c = 9$

$D = (-6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

$\boxed{\text{One Solution, Real and Rational}}$

5. If the roots of $x^2 + bx + 16 = 0$ are equal, then what is the value of b?

$a = 1 \quad b = b \quad c = 16$

The roots are equal when $D = 0$

$D = b^2 - 4ac$

$b^2 - 4ac = 0$

$b^2 - 4(1)(16) = 0$

$b^2 - 64 = 0$

$\quad +64 \quad +64$

$\sqrt{b^2} = \sqrt{64}$

$\boxed{b = \pm 8}$

$b = 8$

$x^2 + 8x + 16 = 0$

$(x+4)(x+4) = 0$

$x + 4 = 0 \qquad x + 4 = 0$

$x = -4 \qquad x = -4$

$b = -8$

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x - 4 = 0 \qquad x - 4 = 0$

$x = 4 \qquad x = 4$

6. If the roots of $ax^2 + 6x + 4 = 0$ are imaginary, then what is the least integral value of a?

The roots are imaginary when $D < 0$

$a = a \quad b = 6 \quad c = 4$

$D = b^2 - 4ac$

$b^2 - 4ac < 0$

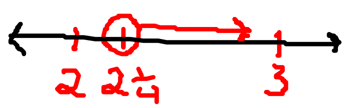
$(6)^2 - 4(a)(4) < 0$

$36 - 16a < 0$

$-36 \qquad -36$

$$\frac{-16a}{-16} < \frac{-36}{-16}$$

$a > \frac{9}{4}$ OR $2\frac{1}{4}$



$\boxed{a = 3}$

7. Find the largest integral value for k for which the roots of $2x^2 + 7x + k = 0$ are real?

The roots are real when $D \geq 0$

$D = b^2 - 4ac$

$b^2 - 4ac \geq 0$

$a = 2 \quad b = 7 \quad c = K$

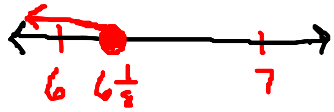
$(7)^2 - 4(2)(K) \geq 0$

$49 - 8K \geq 0$

$-49 \qquad -49$

$$\frac{-8K}{-8} \geq \frac{-49}{-8}$$

$K \leq \frac{49}{8}$ OR $6\frac{1}{8}$



$\boxed{K = 6}$