

Solving Polynomial Equations

Quadratic Equation - An equation of the form $ax^2 + bx + c = 0$, where a , b and c are real numbers and $a \neq 0$.

Methods to Solve Quadratic Equations

1. Factoring
2. Square Root Method
3. Completing the Square
4. Quadratic Formula

Polynomial Equation - An equation of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0 = 0$.

Directions: Find all solutions to each polynomial equation.

1. $20x^3 - 125x = 0$ GCF = $5x$
 $\frac{20x^3}{5x} - \frac{125x}{5x} = 0$

$$5x(4x^2 - 25) = 0$$

$\begin{matrix} \wedge & & \wedge \\ 2x & 2x & 5 & 5 \end{matrix}$

$$5x(2x+5)(2x-5) = 0$$

$$\frac{5x}{5} = 0$$

$$\boxed{x=0}$$

$$\frac{2x+5}{-5} = 0$$

$$\frac{2x}{2} = \frac{-5}{2}$$

$$\boxed{x = -5/2}$$

$$\frac{2x-5}{+5} = 0$$

$$\frac{2x}{2} = \frac{5}{2}$$

$$\boxed{x = 5/2}$$

2. $x^4 - 256 = 0$
 $\begin{matrix} \wedge & & \wedge \\ x^2 & x^2 & 16 & 16 \end{matrix}$

$$(x^2 + 16)(x^2 - 16) = 0$$

$\begin{matrix} \wedge & & \wedge \\ x & x & 4 & 4 \end{matrix}$

$$(x^2 + 16)(x + 4)(x - 4) = 0$$

$$\frac{x^2 + 16}{-16} = 0$$

$$\sqrt{x^2} = \sqrt{-16}$$
$$\boxed{x = \pm 4i}$$

$$\frac{x + 4}{-4} = 0$$

$$\boxed{x = -4}$$

$$\frac{x - 4}{+4} = 0$$

$$\boxed{x = 4}$$

$$3. x^3 - 216 = 0$$

$\begin{array}{c} \uparrow \quad \uparrow \\ x \ x \ x \quad 6 \ 6 \ 6 \end{array}$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$A = x$$

$$B = 6$$

$$(x - 6)(x^2 + 6x + 36) = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x - 6 = 0$$

$$x^2 + 6x + 36 = 0$$

$$+6 \quad +6$$

$$a = 1 \quad b = 6 \quad c = 36$$

$$\boxed{x = 6}$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(36)}}{2(1)} = \frac{-6 \pm \sqrt{-108}}{2} = \frac{-6 \pm \sqrt{-108}}{2} = \frac{-6 \pm 6i\sqrt{3}}{2}$$

$$\boxed{x = -3 \pm 3i\sqrt{3}}$$

$$4. 27x^3 - 64 = 0$$

$\begin{array}{c} \uparrow \quad \uparrow \\ 3 \times 3 \times 3 \times \quad 4 \ 4 \ 4 \end{array}$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$A = 3x \quad B = 4$$

$$(3x - 4)(9x^2 + 12x + 16) = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$3x - 4 = 0$$

$$9x^2 + 12x + 16 = 0$$

$$+4 \quad +4$$

$$a = 9 \quad b = 12 \quad c = 16$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$\boxed{x = \frac{4}{3}}$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(9)(16)}}{2(9)} = \frac{-12 \pm \sqrt{-432}}{18}$$

$$x = \frac{-12 \pm 2i\sqrt{3}}{18} = \frac{-2 \pm 2i\sqrt{3}}{3}$$

$$\begin{array}{c} 432 \\ \uparrow \\ 4 \ 108 \\ \uparrow \\ 36 \ 3 \\ \uparrow \\ 144 \cdot 3 \\ \sqrt{144 \cdot 3} \end{array}$$

$$5. \frac{16x^4}{x^2} - \frac{40x^3}{x^2} + \frac{25x^2}{x^2} = 0 \quad \text{GCF} = x^2$$

$$x^2(16x^2 - 40x + 25) = 0$$

$\begin{array}{ccc} \wedge & & \wedge \\ 4x & 4x & 5 & 5 \end{array}$

$$2(4x \cdot 5) = 40x$$

$$x^2(4x-5)(4x-5) = 0$$

$$\sqrt{x^2} = \sqrt{0}$$

$$4x-5=0 \quad +5 \quad +5$$

$$4x-5=0$$

$$\boxed{x=0}$$

$$\frac{4x}{4} = \frac{5}{4}$$

$$\boxed{x=5/4}$$

$$6. x^3 - 3x^2 + 4x - 12 = 0$$

$$\text{GCF} = x^2 \quad ; \quad \text{GCF} = 4$$

$$\underline{x^2}(x-3) + \underline{4}(x-3) = 0$$

$$\text{GCF} = x-3$$

$$(x-3)(x^2+4) = 0$$

$$x-3=0 \quad x^2+4=0$$

$\begin{array}{cc} +3 & +3 \\ -4 & -4 \end{array}$

$$\boxed{x=3}$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$\boxed{x = \pm 2i}$$

$$7. \frac{x^3 - 6x^2 - 2x + 12}{x^2 - 2} = 0$$

$$\text{GCF} = x^2 \quad \text{GCF} = -2$$

$$\frac{x^2(x-6) - 2(x-6)}{x-6} = 0$$

$$\text{GCF} = x-6$$

$$(x-6)(x^2 - 2) = 0$$

$$x-6=0 \quad x^2-2=0$$

$$+6 \quad +6$$

$$+2 \quad +2$$

$$\boxed{x=6}$$

$$x^2 = 2$$

$$\sqrt{x^2} = \sqrt{2}$$

$$\boxed{x = \pm\sqrt{2}}$$

$$8. x^4 + 5x^2 - 24 = 0$$

$$\overline{3 \cdot 8}$$

$$(x^2 - 3)(x^2 + 8) = 0$$

$$x^2 - 3 = 0$$

$$x^2 + 8 = 0$$

$$+3 \quad +3$$

$$-8 \quad -8$$

$$\sqrt{x^2} = \sqrt{3}$$

$$\sqrt{x^2} = \sqrt{-8}$$

$$\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

$$\boxed{x = \pm\sqrt{3}}$$

$$\boxed{x = \pm 2i\sqrt{2}}$$

9. $36x^4 + 19x^2 - 7 = 0$

1-36
2-18
3-12
4-9
6-6

$$(4x^2 + 7)(9x^2 - 1) = 0$$

$$(9x^2 + 7)(4x^2 - 1) = 0$$

$$(9x^2 + 7)(4x^2 - 1) = 0$$

$$(9x^2 + 7)(2x + 1)(2x - 1) = 0$$

$$9x^2 + 7 = 0 \quad 2x + 1 = 0 \quad 2x - 1 = 0$$

$$\frac{9x^2}{9} = \frac{-7}{9} \quad \frac{2x}{2} = \frac{-1}{2} \quad \frac{2x}{2} = \frac{1}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{-7}{9}} \quad \boxed{x = \frac{-1}{2}} \quad \boxed{x = \frac{1}{2}}$$

$$\boxed{x = \frac{\pm i\sqrt{7}}{3}}$$

10. $x^6 + 9x^3 + 8 = 0$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

$$(x^3 + 1)(x^3 + 8) = 0$$

$$(x+1)(x^2 - x + 1)(x+2)(x^2 - 2x + 4) = 0$$

$$x+1=0 \quad x^2 - x + 1 = 0 \quad x+2=0 \quad x^2 - 2x + 4 = 0$$

$$\boxed{x = -1} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \boxed{x = -2}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$\boxed{x = \frac{1 \pm i\sqrt{3}}{2}}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$\boxed{x = 1 \pm i\sqrt{3}}$$