

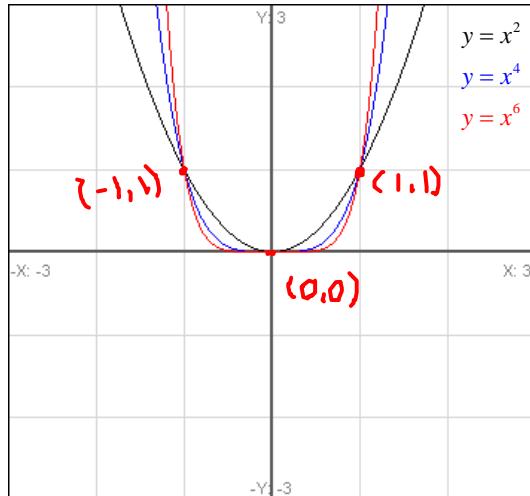
Polynomials of Higher Degree

A polynomial function is a function in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$ where $a \neq 0$ and the coefficients are real numbers.

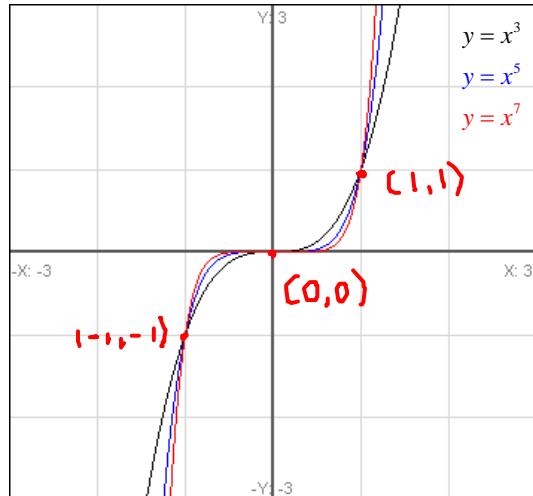
$$\left\{ \begin{array}{l} f(x) = x^5 - 2x^4 + 3x^3 + 5x^2 - x + 1 \\ f(x) = -x^6 + 1 \\ f(x) = \frac{1}{2}(x+1)^4 \end{array} \right.$$

Power Functions

$y = ax^n$ (n is even)

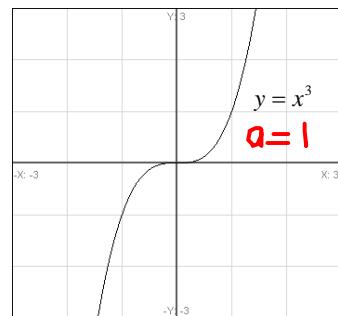
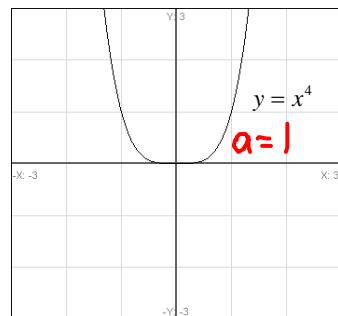


$y = ax^n$ (n is odd)

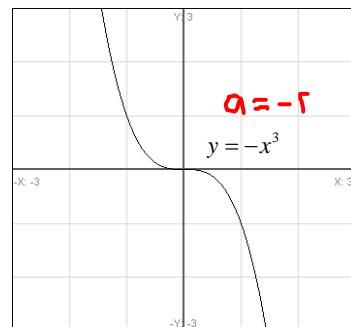
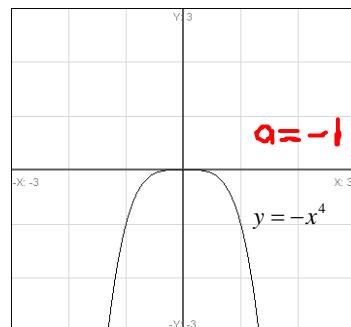


Transformations of Power Functions

$a > 0$

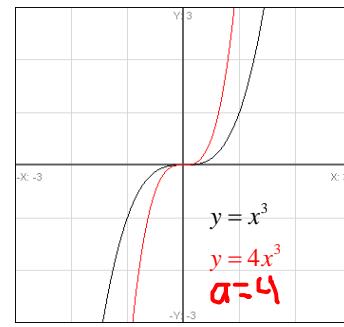
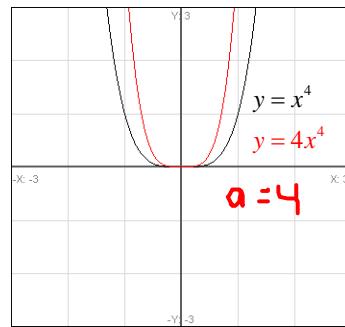


$a < 0$



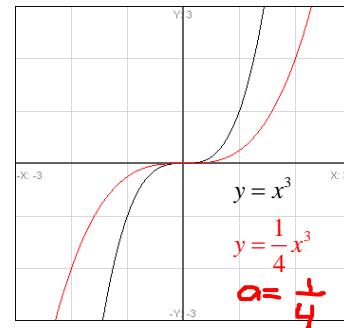
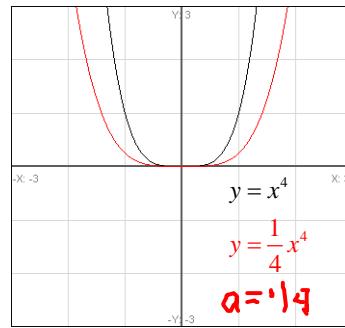
$$|a| > 1$$

Slimmer



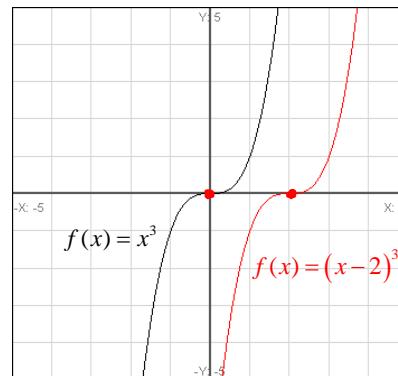
$$0 < |a| < 1$$

Wider



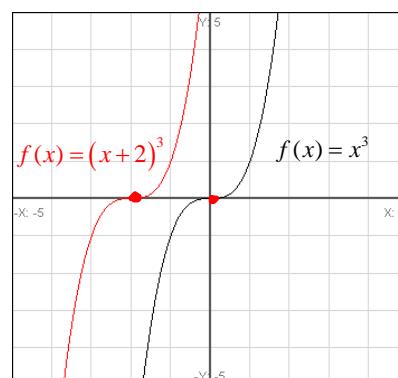
$$f(x) = (x-h)^n$$

Shift graph h units to the right



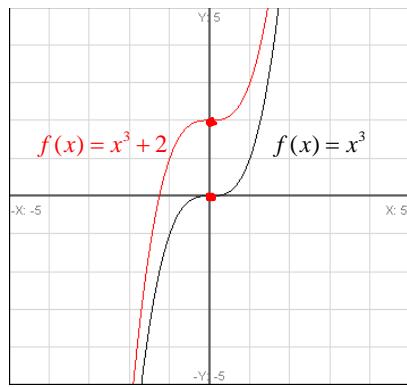
$$f(x) = (x+h)^n$$

Shift graph h units to the left



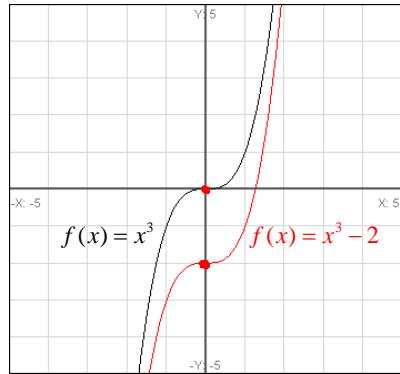
$$f(x) = x^n + k$$

Shift graph k units up



$$f(x) = x^n - k$$

Shift graph k units down

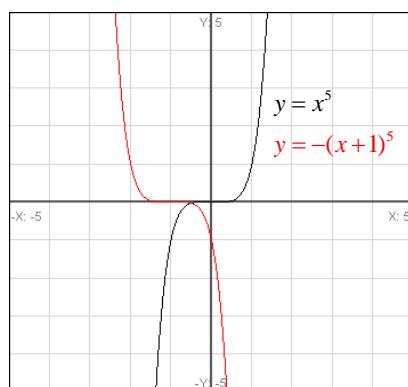


Directions: Sketch a graph of each of the power functions.

1. $f(x) = -(x+1)^5$

$a = -1$

shift graph 1 unit
to the left

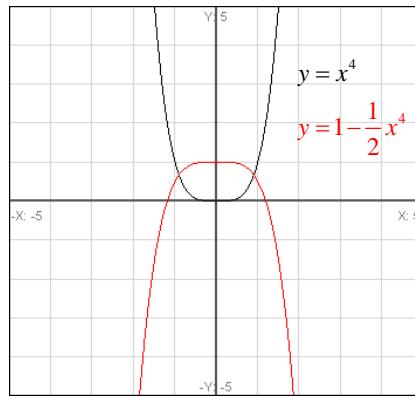


$$2. f(x) = 1 - \frac{1}{2}x^4$$

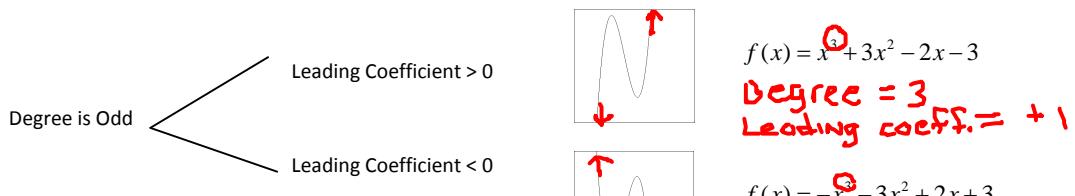
$f(x) = -\frac{1}{2}x^4 + 1$

$\Delta = -\frac{1}{2}$ wider

shift up 1 unit



To Determine the Right and Left Hand Behavior - Look at the sign and the degree of the leading coefficient

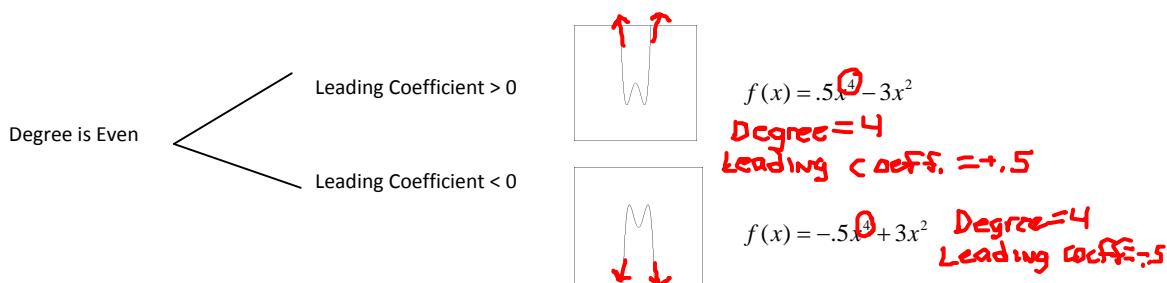


$$f(x) = x^3 + 3x^2 - 2x - 3$$

Degree = 3
Leading coeff. = +1

$$f(x) = -x^3 - 3x^2 + 2x + 3$$

Degree = 3
Leading coeff. = -1



$$f(x) = .5x^4 - 3x^2$$

Degree = 4
Leading coeff. = +.5

$$f(x) = -.5x^4 + 3x^2$$

Degree = 4
Leading coeff. = -.5

Directions: Determine the right and left hand behavior of the graph of the polynomial function.

$$3. f(x) = 2x^5 - 3x + 7$$

Degree = 5
Leading coeff. = +2

$$4. f(x) = -3 + 2x + 5x^2 - 3x^4$$

Degree = 4
Leading coeff. = -3

fall to left, rise to right

fall to left, fall to right

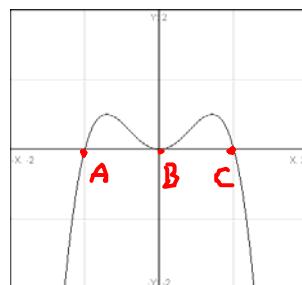
To Determine the x-intercepts/zeros - set the polynomial equal to zero and factor

Graph crosses the x-intercept, multiplicity is odd

A, C

Graph touches the x-intercept, multiplicity is even

B



$$f(x) = -2x^4 + 2x^2$$

$$-2x^4 + 2x^2 = 0$$

$$-2x^2(x^2 - 1) = 0$$

$$-2x^2(x+1)(x-1) = 0$$

$$\frac{-2x^2}{-2} \quad x+1=0 \quad x-1=0$$

$$x=0 \quad -1-1 \quad +1+1$$

$$\sqrt{x^2} = 0 \quad x = -1 \quad x = 1$$

$$x=0 \quad \text{mult.}=1 \quad \text{mult.}=1$$

$$x=0$$

$$\text{mult.}=2$$

Directions: Sketch the graph of each polynomial function. Determine the degree, right and left hand behavior and the zeros of the function.

5. $y = -3x^4 + 3x^2$

Degree: 4

Right/Left Hand Behavior: L.C. = -3



Zeros:

$$-3x^4 + 3x^2 = 0$$

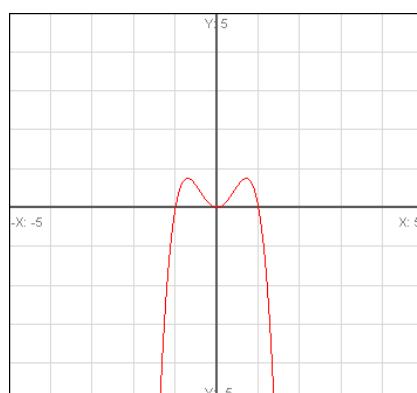
$$-3x^2(x^2 - 1) = 0$$

$$-3x^2(x+1)(x-1) = 0$$

$$\frac{-3x^2}{-3} \quad x+1=0 \quad x-1=0$$

$$x^2=0 \quad x=-1 \quad x=1$$

$$x=0$$



$$6. f(x) = 3x^3 - 15x^2 + 18x$$

Degree: 3

Right/Left Hand Behavior: L.C. = +3

Zeros:

$$\begin{aligned}3x^3 - 15x^2 + 18x &= 0 \\3x(x^2 - 5 + 6) &= 0 \\3x(x - 3)(x - 2) &= 0 \\x = 0 \quad x = 3 \quad x = 2\end{aligned}$$

