Zeros of Polynomial Functions

1. Find the real zeros of the polynomial function.

$f(x) = x^3 - x^2 - 4x + 4$

$x^3 - x^2 - 4x + 4 = 0$

Factor by Grouping

GCF $= x^2$ GCF $= -4$

$x^2(x-1) - 4(x-1) = 0$

$x-1 \quad x-1$

GCF $= x-1$

$(x-1)(x^2-4) = 0$

$(x-1)(x+2)(x-2) = 0$

$x-1=0 \quad x+2=0 \quad x-2=0$

$+1\ +1 \quad -2\ -2 \quad +2\ +2$

$x=1 \quad x=-2 \quad x=2$

Zeros: $x = 1, 2, -2$

2. If $(x+4)$ is a factor of $f(x) = x^3 + 6x^2 + 5x - 12$, find the remaining factors.

$x+4=0$

$-4\ -4$

$x=-4$

-4	1	6	5	-12
	$\downarrow$	-4	-8	12
	$1x^2$	$2x$	-3	0

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

Factors: $(x+4)(x+3)(x-1)$

3. Find the possible rational zeros of $f(x) = 3x^3 + 2x^2 - 3x + 4$.

$\frac{p}{q}$

$p = \pm 1, \pm 2, \pm 4$

$q = \pm 1, \pm 3$

Possible Rational Zeros: $\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}$

$\pm 1, \pm 2, \pm 4$

$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

$= \pm 1, \pm 2, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

4. If $x = 1$ is a zero of $f(x) = 2x^3 - 15x^2 + 34x - 21$, find the remaining zeros.

1	2	-15	34	-21
	↓	2	-13	21
	$2x^2$	$-13x$	21	0

$2x^2 - 13x + 21 = 0$

$2 \cdot 1$ $\qquad$ $1 \cdot 21$, $3 \cdot 7$

$(2x - 7)(x - 3) = 0$

$-7x$, $-6x$

$2x - 7 = 0$
$+7 \quad +7$
$\frac{2x}{2} = \frac{7}{2}$
$x = 7/2$

$x - 3 = 0$
$+3 \quad +3$
$x = 3$

zeros: $1, 3, 7/2$

5. Find the real zeros of $f(x) = x^3 - 31x - 30$.

q $\qquad$ p

$p = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$q = \pm 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

1	1	0	-31	-30
	↓	1	1	-30
	1	1	-30	-60

1 is not a zero

-1	1	0	-31	-30
	↓	-1	1	30
	$1x^2$	$-1x$	-30	0

$x^2 - x - 30 = 0$

$(x - 6)(x + 5) = 0$

$x - 6 = 0$ $\qquad$ $x + 5 = 0$

$x = 6$ $\qquad$ $x = -5$

zeros: $-1, 6, -5$

6. Write a polynomial function that has the given zeros.

a) -5, -1, 2

$x=-5 \quad x=-1 \quad x=2$

$(x+5)(x+1)(x-2)$

FOIL

$x^2-2x+x-2$

$(x+5)(x^2-x-2)$

$x^3-x^2-2x+5x^2-5x-10$

$f(x)=x^3+4x^2-7x-10$

b) -1, 2, 3*i*

$x=-1 \quad x=2 \quad x=3i \quad x=-3i$

$(x+1)(x-2) \quad (x-3i)(x+3i)$

$x^2-2x+x-2$

(x^2-x-2)

$x^2+3ix-3ix-9i^2$

$x^2-9(-1)$

(x^2+9)

$(x^2-x-2)(x^2+9)$

$x^4+9x^2-x^3-9x-2x^2-18$

$f(x)=x^4-x^3+7x^2-9x-18$