Solving Rational Equations

Step 1: Factor all denominators and find restrictions.
Step 2: Find the least common denominator (LCD).
Step 3: Multiply each fraction by what is missing in the LCD.
Step 4: Cancel all denominators.
Step 5: Solve for the variable and check your answer.

Directions: Find all solutions of each rational equation.

1. $3-\frac{4}{x}=\frac{5}{2}$ $x\neq 0$

$\frac{2x\cdot 3}{2x\cdot 1}-\frac{2\cdot 4}{2\cdot x}=\frac{5}{2}\cdot x$

LCD = 2x

$\frac{6x}{2x}-\frac{8}{2x}=\frac{5x}{2x}$

$6x-8=5x$
$-5x \quad -5x$
$x-8=0$
$+8 \quad +8$
$\boxed{x=8}$

2. $\frac{5}{3x}+\frac{3}{x}=1$ $3x\neq 0$ $x\neq 0$

$\frac{3x}{3}\neq\frac{0}{3}$

$x\neq 0$

$\frac{5}{3x}+\frac{3\cdot 3}{3\cdot x}=\frac{1\cdot 3x}{1\cdot 3x}$

LCD = 3x

$\frac{5}{3x}+\frac{9}{3x}=\frac{3x}{3x}$

$5+9=3x$
$\frac{14}{3}=\frac{3x}{3}$
$\boxed{x=14/3}$

3. $\frac{x-5}{x+3} = \frac{1}{5}$

$5(x-5) = 1(x+3)$

$5x - 25 = x + 3$

$-x \qquad -x$

$4x - 25 = 3$

$+25 \quad +25$

$\frac{4x}{4} = \frac{28}{4}$

$\boxed{x = 7}$

4. $\frac{3(x-6)}{5} = \frac{4(x+2)}{3}$

$3 \cdot 3(x-6) = 5 \cdot 4(x+2)$

$9(x-6) = 20(x+2)$

$9x - 54 = 20x + 40$

$-9x \qquad -9x$

$-54 = 11x + 40$

$-40 \qquad -40$

$\frac{-94}{11} = \frac{11x}{11}$

$\boxed{x = -\frac{94}{11} \text{ OR } -8\frac{6}{11}}$

5. $\frac{3a-2}{2a+2} = \frac{3}{a-1}$

$3(2a+2) = (a-1)(3a-2)$

$6a + 6 = 3a^2 - 2a - 3a + 2$

$6a + 6 = 3a^2 - 5a + 2$

$-6a + 6 \qquad -6a - 6$

$0 = 3a^2 - 11a - 4$

$3a + 1 = 0$

$-1 \quad -1$

$\frac{3a}{3} = \frac{-1}{3}$

$\boxed{a = -\frac{1}{3}}$

$a - 4 = 0$

$+4 \quad +4$

$\boxed{a = 4}$

$\frac{3a^2 - 11a - 4 = 0}{1 \cdot 3 \qquad 1 \cdot 4 \quad 2 \cdot 2}$

$(3a + 1)(a - 4) = 0$

$1a$

$-12a$

6. $\frac{1}{x+3}+\frac{1}{x-3}=\frac{3}{x^2-9}$

$x+3\neq 0$ $\quad x\neq -3$ $\qquad x-3\neq 0$ $\quad x\neq 3$

$\frac{(x-3)\cdot 1}{(x-3)(x+3)}+\frac{1\cdot(x+3)}{(x-3)(x+3)}=\frac{3}{(x+3)(x-3)}$

LCD $=(x+3)(x-3)$

$\frac{1(x-3)}{(x-3)(x+3)}+\frac{1(x+3)}{(x-3)(x+3)}=\frac{3}{(x+3)(x-3)}$

$x-3+x+3=3$

$\frac{2x}{2}=\frac{3}{2}$

$\boxed{x=3/2}$

7. $\frac{3y-2}{y+1}=4-\frac{y+2}{y-1}$

$y+1\neq 0$ $\quad y\neq -1$ $\qquad y-1\neq 0$ $\quad y\neq 1$

$\frac{(y-1)\cdot 3y-2}{(y-1)(y+1)}=\frac{4\cdot(y+1)(y-1)}{1\cdot(y+1)(y-1)}-\frac{y+2\cdot(y+1)}{y-1\cdot(y+1)}$

LCD $=(y+1)(y-1)$

$\frac{(y-1)(3y-2)}{(y-1)(y+1)}=\frac{4(y+1)(y-1)}{(y+1)(y-1)}-\frac{(y+2)(y+1)}{(y+1)(y-1)}$

$(y-1)(3y-2)=4(y+1)(y-1)-(y+2)(y+1)$

$3y^2-2y-3y+2=4(y^2-1)-(y^2+y+2y+2)$

$3y^2-5y+2=4y^2-4-y^2-y-2y-2$

$3y^2-5y+2=3y^2-3y-6$

$-3y^2 \qquad -3y^2$

$-5y+2=-3y-6$

$+3y \qquad +3y$

$-2y+2=-6$

$-2 \qquad -2$

$\frac{-2y}{-2}=\frac{-8}{-2}$

$\boxed{y=4}$

8. $\frac{2x}{x+2}=\frac{x}{x+3}-\frac{3}{x^2+5x+6}$

$x+2\neq 0$ $\quad x\neq -2$ $\qquad$ $x+3\neq 0$ $\quad x\neq -3$

$(x+3)\cdot\frac{2x}{x+2}=\frac{x\cdot(x+2)}{x+3\cdot(x+2)}-\frac{3}{(x+2)(x+3)}$

LCD $=(x+2)(x+3)$

$\frac{2x(x+3)}{(x+3)(x+2)}=\frac{x(x+2)}{(x+3)(x+2)}-\frac{3}{(x+3)(x+2)}$

$2x^2+6x=x^2+2x-3$

$-x^2-2x+3\quad -x^2-2x+3$

$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x+3=0$ $\qquad$ $x+1=0$

$x=-3$ $\qquad$ $\boxed{x=-1}$