

Word Problems - Rational Equations

1. The area of Frank's garden is 60 square feet. Determine the length and width if the width is 5 feet less than $\frac{3}{5}$ the length.

$$A = 60 \text{ sq. ft.}$$

$$A = L \times W$$

$$L = x = \boxed{15 \text{ feet}}$$

$$W = \frac{3}{5}x - 5$$

$$= \frac{3}{5} \cdot \frac{15}{1} - 5$$

$$= 9 - 5 = \boxed{4 \text{ feet}}$$

$$3x + 20 = 0 \quad x - 15 = 0$$

$$\begin{array}{r} -20 \\ \hline 3x = -20 \\ \frac{3x}{3} = \frac{-20}{3} \\ x = \frac{-20}{3} \end{array} \quad \begin{array}{r} +15 \\ \hline x = 15 \end{array}$$

$$A = L \times W$$

$$60 = x \left(\frac{3}{5}x - 5 \right)$$

$$60 = \frac{3}{5}x^2 - 5x$$

$$\frac{5 \cdot 60}{5 \cdot 1} = \frac{3x^2}{5} - \frac{5x \cdot 5}{1 \cdot 5} \quad \text{LCD} = 5$$

$$\frac{300}{5} = \frac{3x^2}{5} - \frac{25x}{5}$$

$$300 = 3x^2 - 25x$$

$$\begin{array}{r} -200 \\ \hline 300 = 3x^2 - 25x \\ -200 \qquad \qquad -300 \end{array}$$

$$3x^2 - 25x - 300 = 0$$

1.300	4.75	12.25
2.150	5.60	15.20
3.100	10.30	

$$(3x + 20)(x - 15) = 0$$

$$\begin{array}{c} \underbrace{\hspace{10em}}_{45x} \\ \underbrace{\hspace{10em}}_{20x} \end{array}$$

2. One number is 3 times another. The sum of their reciprocals is $\frac{7}{12}$. Find the numbers.

$$1^{\text{st}} \text{ number} = \frac{x}{1}$$

$$2^{\text{nd}} \text{ number} = \frac{3x}{1}$$

$$12 \cdot \frac{1}{12 \cdot x} + 4 \cdot \frac{1}{4 \cdot 3x} = \frac{7 \cdot x}{12 \cdot x}$$

$$\text{LCD} = 12x$$

$$1^{\text{st}} \text{ number} = x = \boxed{\frac{16}{7}}$$

$$2^{\text{nd}} \text{ number} = 3x = \frac{3 \left(\frac{16}{7} \right)}{1} = \boxed{\frac{48}{7}}$$

$$x = \frac{16}{7}$$

3. The current in a river is 2 miles per hour. It takes Katie the same amount of time to travel 10 miles downstream as 2 miles upstream. Determine the speed at which Katie would travel in still water.

$D = R \cdot T$ Distance = Rate * Time

Current = 2 mph

$R = 3 \text{ mph}$

Upstream

$D = 2 \text{ m}$

$T = t$

$R = R - 2$

$D = R \cdot T$

$2 = (R - 2)t$

$t = \frac{2}{R - 2}$

Downstream

$D = 10 \text{ m}$

$T = t$

$R = R + 2$

$D = R \cdot T$

$10 = (R + 2)t$

$t = \frac{10}{R + 2}$

$\frac{2}{R - 2} = \frac{10}{R + 2}$

$10(R - 2) = 2(R + 2)$

$10R - 20 = 2R + 4$

$-2R - 20 - 2R + 20$

$8R = 24 \quad R = 3 \text{ mph}$

4. It takes Michael 30 hours to paint a house. Mary can paint the same house by herself in 20 hours. How long will it take them to paint the house if they work together?

$D = R \cdot T$
Amount = $R \cdot T$
1

Work : $t = 12 \text{ h}$
together

Michael

$T = 30 \text{ h}$

$R = \frac{1}{30}$

$A = R \cdot T$

$A = \frac{1}{30} t$

Mary

$T = 20 \text{ h}$

$R = \frac{1}{20}$

$A = R \cdot T$

$A = \frac{1}{20} t$

$\frac{1}{30} t + \frac{1}{20} t = 1$

$2 \cdot \frac{t}{30} + \frac{t \cdot 3}{20 \cdot 3} = \frac{1 \cdot 60}{1 \cdot 60}$

LCD = 60

$\frac{2t}{60} + \frac{3t}{60} = \frac{60}{60}$
 $2t + 3t = 60$

$\frac{5t}{5} = \frac{60}{5}$
 $t = 12 \text{ h}$

5. One hose can fill a pool in 12 hours and another hose can empty the same pool in 15 hours. If one hose is filling the pool while the other hose is emptying the pool, how long will it take to fill the empty pool?

$$D = R \cdot T$$

$$A = R \cdot T$$

1

Fill

$$T = 12 \text{ h}$$

$$R = \frac{1}{12}$$

Empty

$$T = 15 \text{ h}$$

$$R = \frac{1}{15}$$

Fill: $t = 60 \text{ h}$
 pool: $t = 60 \text{ h}$

$$A = R \cdot T$$

$$A = R \cdot T$$

$$A = \frac{1}{12} t$$

$$A = \frac{1}{15} t$$

$$\frac{1}{12} t - \frac{1}{15} t = 1$$

$$\frac{5 \cdot t}{5 \cdot 12} - \frac{t \cdot 4}{15 \cdot 4} = \frac{1 \cdot 60}{1 \cdot 60} \quad \text{LCD} = 60$$

$$\frac{5t}{60} - \frac{4t}{60} = \frac{60}{60}$$

$$5t - 4t = 60$$

$$t = 60 \text{ hours}$$

6. When Steve cleans all the windows in a building it takes him 8 hours. When Nicole and Steve work together, they can clean all the windows in 5 hours. How long does it take Nicole to clean all the windows by herself?

$A = R \cdot T$

Steve
 $T = 8 \text{ h}$
 $R = \frac{1}{8}$
 $A = \frac{1}{8}t$

Nicole
 $t = 13\frac{1}{3}$
 hours

Steve and Nicole
 $T = 5 \text{ h}$

Steve + Nicole = 1

$\frac{1}{8}t$ + $\square t$ = 1

$\frac{1}{8}(5) + \frac{1}{x}(5) = 1$

$\frac{x \cdot 5}{x \cdot 8} + \frac{8 \cdot 5}{8 \cdot x} = \frac{1 \cdot 8x}{1 \cdot 8x}$ LCD = 8x

$\frac{5x}{8x} + \frac{40}{8x} = \frac{8x}{8x}$

$5x + 40 = 8x$
 $-5x \quad -5x$

$\frac{40}{3} = \frac{3x}{3}$

$x = 13\frac{1}{3} \text{ h}$