

Solving Equations with Rational Exponents

$$\frac{a}{x^b} \begin{array}{l} \rightarrow \text{power} \\ \rightarrow \text{root} \end{array}$$

$$27^{\frac{2}{3}} = \sqrt[3]{27} = (3)^2 = \boxed{9} \quad (-8)^{\frac{5}{3}} = \sqrt[3]{-8} = (-2)^5 = \boxed{-32}$$

Directions: Solve each equation.

1. $x^{\frac{3}{2}} - 16 = 0$

$$+16 +16$$

$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (16)^{\frac{2}{3}}$$

$$x = 16^{\frac{3}{2}}$$

$$\sqrt{16} = \pm 4$$

$$(4)^3 = 64$$

$$(-4)^3 = -64$$

$$\boxed{x = \pm 64}$$

Root = 3 (odd)
No check required

2. $(x+4)^{\frac{4}{3}} = 81$

$$\left[(x+4)^{\frac{4}{3}}\right]^{\frac{3}{4}} = (81)^{\frac{3}{4}}$$

$$x+4 = (81)^{\frac{3}{4}}$$

$$\sqrt[4]{81} = \pm 3$$

$$(3)^3 = 27$$

$$(-3)^3 = -27$$

$$x+4 = 27$$

$$x+4 = -27$$

$$-4 \quad -4$$

$$-4 \quad -4$$

$$\boxed{x = 23}$$

$$\boxed{x = -31}$$

$$3. (2x+6)^{\frac{3}{2}} = -125$$

$$\left[(2x+6)^{\frac{3}{2}} \right]^{\frac{2}{3}} = (-125)^{\frac{2}{3}}$$

Root = 2 (Even)
check required

$$2x+6 = (-125)^{\frac{2}{3}} \quad \sqrt[3]{-125} = (-5)^2 = 25$$

$$2x+6 = 25$$

$$-6 \quad -6$$

$$\frac{2x}{2} = \frac{19}{2}$$

$$x = 9\frac{1}{2}$$

check

$$x = 9\frac{1}{2} \quad (2x+6)^{\frac{3}{2}} = -125$$

$$\left[2\left(9\frac{1}{2}\right) + 6 \right]^{\frac{3}{2}} = -125$$

$$[25]^{\frac{3}{2}} = -125$$

$$\sqrt[2]{25} = (5)^2 = 125$$

$$125 = -125 \quad \times$$

no solution

$$4. x^{\frac{2}{3}} - 3x^{\frac{1}{3}} = -2$$

$$+2 \quad +2$$

$$x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 2 = 0$$

$$\frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$$

$$x^2 - 3x + 2 = 0$$

$$+2 \quad +2$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x-2=0 \quad x-1=0$$

$$x=2 \quad x=1$$

$$\left(x^{\frac{1}{3}} - 2 \right) \left(x^{\frac{1}{3}} - 1 \right) = 0$$

$$x^{\frac{1}{3}} - 2 = 0$$

$$x^{\frac{1}{3}} - 1 = 0$$

$$\left(x^{\frac{1}{3}} \right)^3 = (2)^3$$

$$\left(x^{\frac{1}{3}} \right)^3 = (1)^3$$

$$x = 2^3$$

$$x = 1^3$$

$$\boxed{x=8}$$

$$\boxed{x=1}$$

root = 3 (odd)

no check
required

$$5. -2x - 5x^{\frac{1}{2}} + 3 = 0$$

root = 2 (even)
check required

$$(-2x+3)^{\frac{2}{2}} = (5x^{\frac{1}{2}})^{\frac{2}{2}}$$

$$\begin{aligned} (-2x+3)^2 &= 25x \\ (-2x+3)(-2x+3) &= 25x \\ 4x^2 - 6x - 6x + 9 &= 25x \\ 4x^2 - 12x + 9 &= 25x \\ -25x & \quad -25x \end{aligned}$$

$$4x^2 - 37x + 9 = 0$$

$$\begin{array}{r} 4-1 \\ 2 \cdot 2 \end{array} \quad \begin{array}{r} 1-9 \\ 3 \cdot 3 \end{array}$$

$$(4x - 1)(x - 9) = 0$$

$$\begin{array}{c} \underbrace{\hspace{1.5cm}} \\ 36x \end{array}$$

$$\begin{array}{cc} 4x-1=0 & x-9=0 \\ +1 & +9 \\ \hline 4x=1 & x=9 \\ \frac{4}{4} & \end{array}$$

$$x = \frac{1}{4}$$

$$x = \frac{1}{4}$$

check

$$x = \frac{1}{4}$$

$$\begin{aligned} -2x - 5x^{\frac{1}{2}} + 3 &= 0 \\ -2\left(\frac{1}{4}\right) - 5\left(\frac{1}{4}\right)^{\frac{1}{2}} + 3 &= 0 \end{aligned}$$

$$\begin{aligned} -\frac{1}{2} - \frac{5}{2} + 3 &= 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$$x = 9$$

$$-2x - 5x^{\frac{1}{2}} + 3 = 0$$

$$-2(9) - 5(9)^{\frac{1}{2}} + 3 = 0$$

$$-18 - 15 + 3 = 0$$

$$-30 = 0 \quad \times$$

$$6. x^{\frac{2}{5}} - 5x^{\frac{1}{5}} + 6 = 0$$

root = 5 (odd)
no check required

$$(x^{\frac{1}{5}} - 2)(x^{\frac{1}{5}} - 3) = 0$$

$$\begin{array}{cc} x^{\frac{1}{5}} - 2 = 0 & x^{\frac{1}{5}} - 3 = 0 \\ +2 & +3 \\ \hline x^{\frac{1}{5}} = 2 & x^{\frac{1}{5}} = 3 \end{array}$$

$$\left(x^{\frac{1}{5}}\right)^5 = (2)^5 \quad \left(x^{\frac{1}{5}}\right)^5 = (3)^5$$

$$x = 2^5$$

$$x = 3^5$$

$$x = 32$$

$$x = 243$$

$$7. \frac{2x^2}{2 \cdot 1} - \frac{7x^1}{1 \cdot 6} + 6 = 0$$

root = 3 (odd)
 No check
 required

$$(2x^{\frac{1}{3}} - 3)(x^{\frac{1}{3}} - 2) = 0$$

$$2x^{\frac{1}{3}} - 3 = 0$$

$$x^{\frac{1}{3}} - 2 = 0$$

$$(2x^{\frac{1}{3}})^3 = (3)^3$$

$$(x^{\frac{1}{3}})^3 = (2)^3$$

$$\frac{8x}{8} = \frac{27}{8}$$

$$x = 8$$

$$x = \frac{27}{8}$$