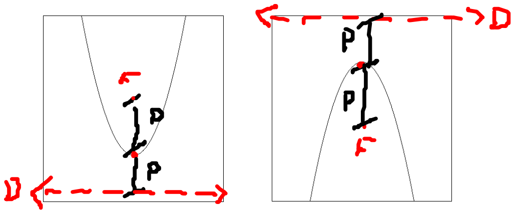
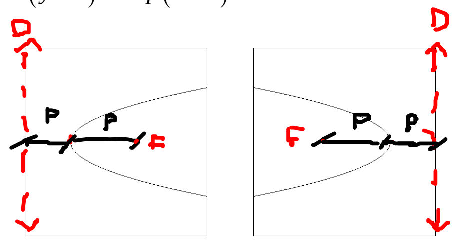
Conic Sections - Parabolas

<u>Standard Form of the Equation of a Parabola</u>

Vertical Parabola	Horizontal Parabola
$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$





Vertex $= (h, k)$

Focus $=$ p units from the vertex (inside the parabola)

Directrix $=$ p units from the vertex (outside the parabola)

Length of Focal Chord $= 4p$

1. Find the vertex, focus and directrix of the parabola and sketch its graph.

a) $x^2 + 8y = 0$

vertical parabola

$(x-h)^2 = 4p(y-k)$

$x^2 + 8y = 0$

$-8y \quad -8y$

$x^2 = \underbrace{-8}_{4p}y$

$\frac{4p}{4} = \frac{8}{4}$

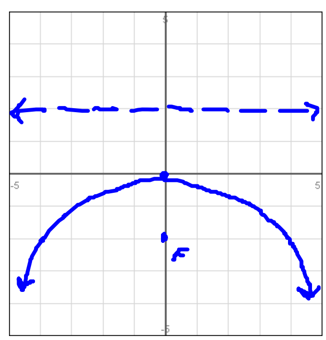
$p = 2$

vertex $(0,0)$

Focal chord $= 4p$

$= 4(2)$

$= 8$



D: $\boxed{y = 2}$

$\boxed{V: (0,0)}$

$\boxed{F: (0,-2)}$

b) $x^2 - 2x - 4y + 9 = 0$

$+4y \ -9$

vertical

$(x-h)^2 = 4p(y-k)$

$x^2 - 2x + 1 = 4y - 9 + 1$

$\frac{2}{2} = (1)^2 = 1$

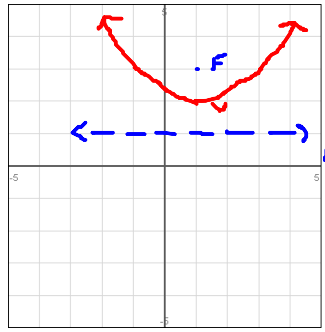
$x^2 - 2x + 1 = 4y - 8$

$(x-1)^2 = \underbrace{4}_{4p}(y-2)$

vertex: $(1, 2)$

$4p = 4$

$p = 1$



$\boxed{F: (1, 3)}$

$\boxed{V: (1, 2)}$

$\boxed{D: y = 1}$

Focal chord

$4p = 4(1) = 4$

c) $y^2 + 6y + 2x + 25 = 0$

$-2x - 25$

Horizontal

$(y-k)^2 = 4p(x-h)$

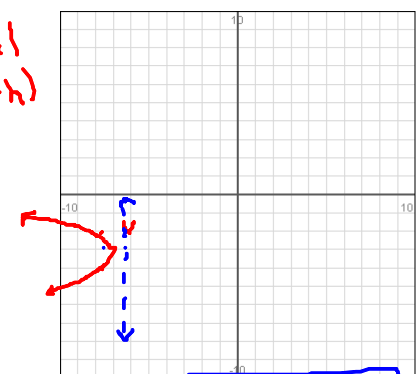
$y^2 + 6y + 9 = -2x - 25 + 9$

$\frac{6}{2} = (3)^2 = 9$

$y^2 + 6y + 9 = -2x - 16$

$(y+3)^2 = \underbrace{-2}_{4p}(x+8)$

vertex: $(-8, -3)$



Focal chord

$4p = 4(\frac{1}{2}) = 2$

$\frac{4p}{4} = \frac{2}{4}$

$p = 1/2$

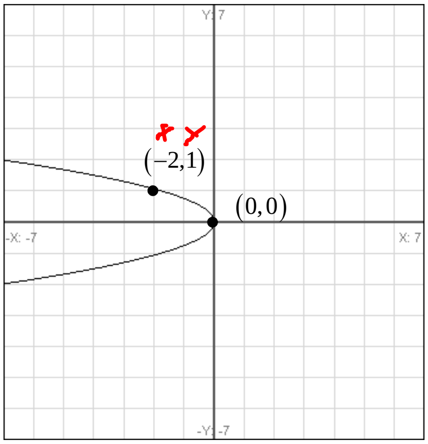
$\boxed{D: x = -7\frac{1}{2}}$

$\boxed{V: (-8, -3)}$

$\boxed{F: (-8\tfrac{1}{2}, -3)}$

2. Find the standard form of the equation of the parabola.

a)



$(y-k)^2 = 4p(x-h)$

V $(0,0)$

$y^2 = 4px$

$(1)^2 = 4p(-2)$

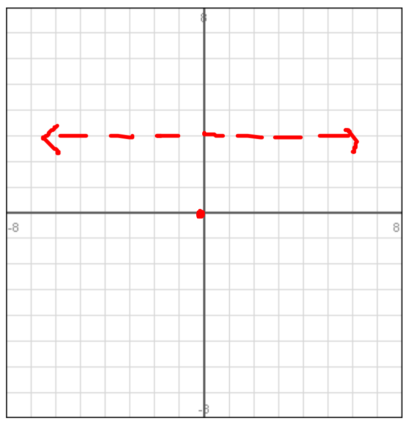
$\frac{1}{-8} = \frac{-8p}{-8}$

$p = -\frac{1}{8}$

$y^2 = 4\left(-\frac{1}{8}\right)x$

$\boxed{y^2 = -\frac{1}{2}x}$

b) Vertex is at the origin.
Directrix: $y = 3$



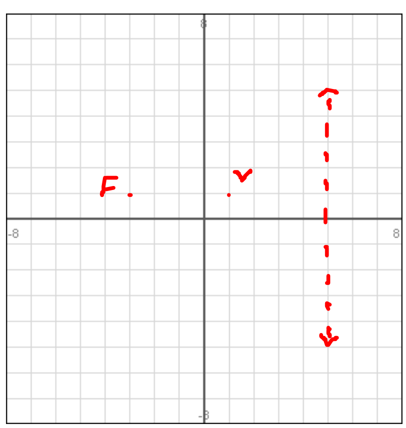
$(x-h)^2 = 4p(y-k)$

V: $(0,0)$

$p = 3$

$\boxed{x^2 = -12y}$

c) Focus: $(-3,1)$
Directrix: $x = 5$



V: $(1,1)$

$(y-k)^2 = 4p(x-h)$

$p = 4$

$\boxed{(y-1)^2 = -16(x-1)}$

3. The equation of a parabola and the tangent line are given. Find the coordinates of the point of tangency.

$x^2 + 12y = 0$

$x + y = 3$

$x^2 + 12y = 0$
$-12y \quad -12y$

$x^2 = -\underbrace{12}_{4p}y$

V: (0,0)

$4p = 12$
$p = 3$

Focal chord
$4p = 4(3) = 12$

$x + y = 3$
$-x \quad -x$

$y = -x + 3$

m = -1   b: (0,3)

$x^2 + 12(-x + 3) = 0$

$x^2 - 12x + 36 = 0$

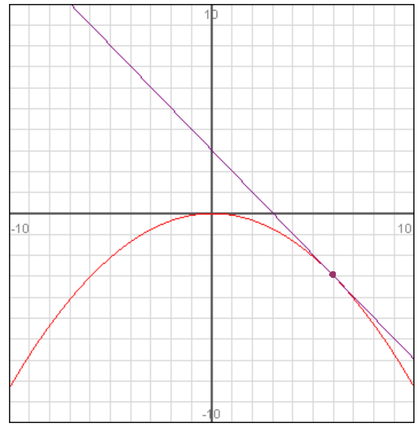
$(x-6)(x-6) = 0$

$x = 6$

$y = -6 + 3$

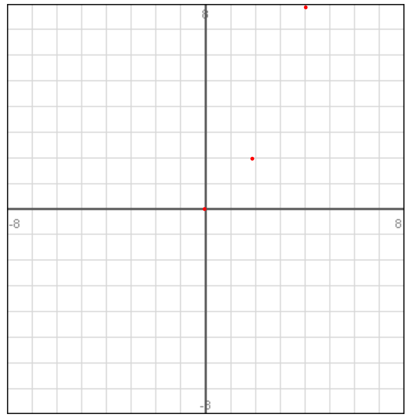
$y = -3$

$\boxed{(6, -3)}$



4. Find the equation of the parabola that contains the points $\overset{x\ y}{(0,0)}$, $\overset{x\ y}{(2,2)}$ and $\overset{x\ y}{(4,8)}$.

$(x-h)^2 = 4p(y-k)$



(0,0): $(0-h)^2 = 4p(0-k)$

* $h^2 = -4pk$

(2,2): $(2-h)^2 = 4p(2-k)$

$(2-h)(2-h) = 4p(2-k)$

$4-2h-2h+h^2 = 8p-4pk$

* $4-4h+h^2 = 8p-4pk$

(4,8) $(4-h)^2 = 4p(8-k)$

$(4-h)(4-h) = 4p(8-k)$

$16-4h-4h+h^2 = 32p-4pk$

* $16-8h+h^2 = 32p-4pk$

$4-4h+\cancel{h^2} = 8p+\cancel{h^2}$

$4-4h = 8p$ *

Solve for h

$\frac{-4h}{-4} = \frac{8p}{-4} \frac{-4}{-4}$

$h = -2p+1$

$h = -2\left(\frac{1}{2}\right)+1$

$h = -1+1 = 0$ $\boxed{h=0}$

$h^2 = -4pk$

$0^2 = -4\left(\frac{1}{2}\right)k$

$0 = -2k$

$\boxed{k=0}$

$16-8h+\cancel{h^2} = 32p+\cancel{h^2}$

$16-8h = 32p$ *

$16-8(-2p+1) = 32p$

$16+16p-8 = 32p$

$-16p \qquad -16p$

$\frac{8}{16} = \frac{16p}{16}$

$\boxed{p = 1/2}$

$x^2 = 4\left(\frac{1}{2}\right)y$

$\boxed{x^2 = 2y}$