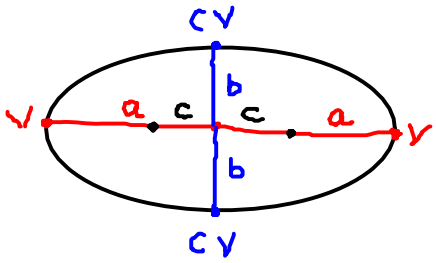
Conic Sections - Ellipses

Standard Form for the Equation of an Ellipse

Horizontal Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



* $a > b$

Center $= (h,k)$

Major Axis $= 2a$

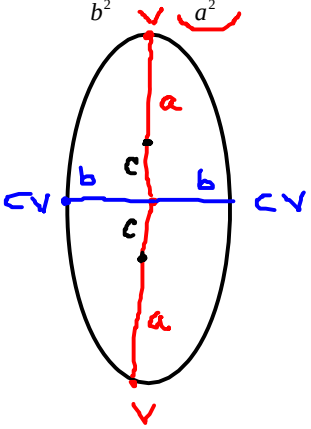
Minor Axis $= 2b$

Foci: $c^2 = a^2 - b^2$

Eccentricity: $e = \frac{c}{a}$

Vertical Ellipse

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

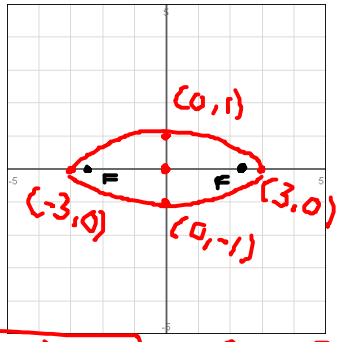


1. Find the center, vertices, foci and lengths of the major and minor axes.

a) $x^2+9y^2=9$

$\frac{x^2}{9}+\frac{9y^2}{9}=\frac{9}{9}$

$\frac{x^2}{9}+\frac{y^2}{1}=1$



horizontal

$a^2=9 \quad a=3$

$b^2=1 \quad b=1$

center (0,0)

Vertices: (3,0)(-3,0)

Co-Vertices: (0,1)(0,-1)

Major Axis = 6

Minor Axis = 2

Foci $(2\sqrt{2},0)(-2\sqrt{2},0)$

$c^2 = a^2 - b^2$

$c^2 = 9 - 1$

$c^2 = 8$

$c = \sqrt{8}$

$c = 2\sqrt{2}$

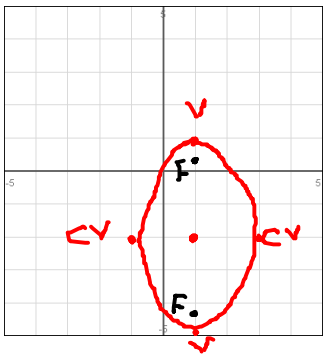
b) $9x^2+4y^2-18x+16y-11=0$

$+11 \quad +11$

$9x^2-18x+4y^2+16y=11$

$9(x^2-2x+1)+4(y^2+4y+4)=11+9+16$

$\frac{2}{2}=(1)^2=1 \qquad \frac{4}{2}=(2)^2=4$



major = 6
minor = 4

V: (1,1)(1,-5)

$9(x^2-2x+1)+4(y^2+4y+4)=36$

$\frac{9(x-1)^2}{36}+\frac{4(y+2)^2}{36}=\frac{36}{36}$

$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$

center (1,-2)

$a^2=9 \quad a=3$

$b^2=4 \quad b=2$

CV: (3,-2) (-1,-2)

Foci: $c^2=a^2-b^2$

$c^2=9-4$

$c^2=5$

$c=\sqrt{5}$

F: $(1,-2+\sqrt{5})$ $(1,-2-\sqrt{5})$

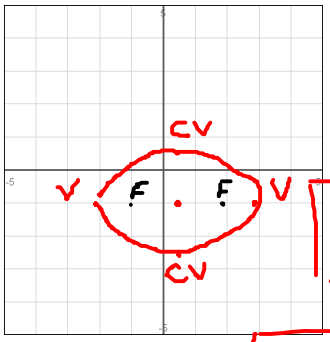
c) $12x^2+20y^2-12x+40y-37=0$

$+37 \quad +37$

$12x^2-12x+20y^2+40y=37$

$12(x^2-x+\frac{1}{4})+20(y^2+2y+1)=37+3+20$

$\frac{1}{2}\div\left(\frac{1}{2}\right)^2=\frac{1}{4} \qquad \frac{2}{2}=(1)^2=1$



$c^2=5 \quad a=\sqrt{5}$

$b^2=3 \quad b=\sqrt{3}$

Major $=2\sqrt{5}$

Minor $=2\sqrt{3}$

$\frac{12(x-1/2)^2}{60}+\frac{20(y+1)^2}{60}=\frac{60}{60}$

$\frac{(x-1/2)^2}{5}+\frac{(y+1)^2}{3}=1$

center $(1/2,-1)$

V: $(1/2+\sqrt{5},-1)$ $(1/2-\sqrt{5},-1)$

CV $(1/2,-1+\sqrt{3})$ $(1/2,-1-\sqrt{3})$

F: $c^2=a^2-b^2$

$c^2=5-3$

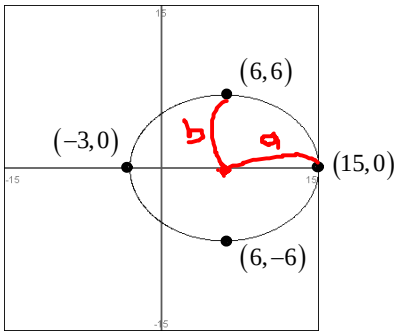
$c^2=2$

$c=\sqrt{2}$

$(1/2+\sqrt{2},-1)$ $(1/2-\sqrt{2},-1)$

2. Find the standard form of the equation of the ellipse.

a)



$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Center $(6,0)$

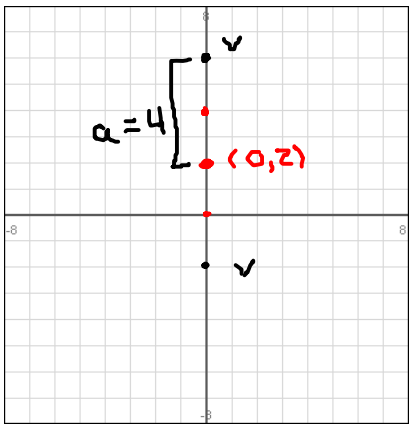
$a = 9$

$b = 6$

$$\frac{(x-6)^2}{81} + \frac{y^2}{36} = 1$$

b) Foci: $(0,0)$ and $(0,4)$

Length of Major Axis: 8



$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Center $(0,2)$

$a^2 = 16$

$a = 4 \quad c = 2$

$c^2 = a^2 - b^2$

$2^2 = 4^2 - b^2$

$4 = 16 - b^2$

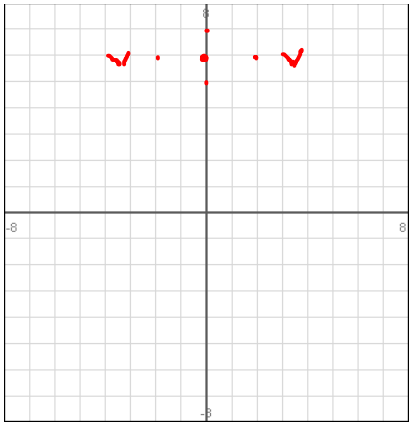
$-16 \quad -16$

$-12 = -b^2$

$b^2 = 12$

$$\frac{x^2}{12} + \frac{(y-2)^2}{16} = 1$$

c) Vertices: $(-2,6)$ and $(2,6)$
Length of Minor Axis: 2



center $(0,6)$

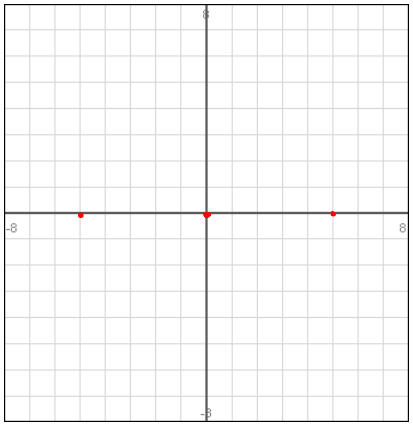
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$a=2 \qquad b=1$

$\frac{x^2}{4} + \frac{(y-6)^2}{1} = 1$

d) Vertices: $(-5,0)$ and $(5,0)$

$e = \frac{3}{5}$ $\quad e = \frac{c}{a}$



center $(0,0)$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$a=5 \qquad c=3$

$c^2 = a^2 - b^2$
$3^2 = 5^2 - b^2$
$9 = 25 - b^2$
$-25 \quad -25$
$-16 = -b^2 \qquad b^2 = 16$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$