The Natural Logarithmic Function

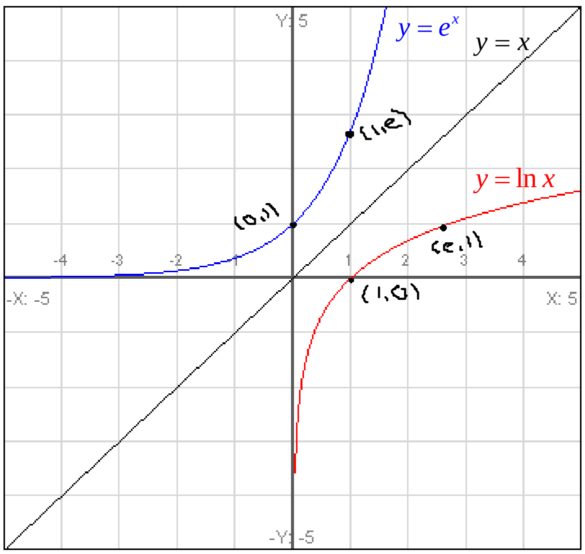
The irrational number $e \approx 2.71828...$ is called the natural base. It represents the number whose natural log is 1.

Natural Exponential Function

$$y = e^x$$

Natural Logarithmic Function

$$y = \log_e x \qquad y = \ln x$$



Properties of the Natural Exponential Function
Domain: All Real Numbers
Range: $y > 0$
y-intercept: $(0, 1)$
Contains the point $(1, e)$
Horizontal Asymptote: $y = 0$
The function is increasing.

Properties of the Natural Logarithmic Function
Domain: $x > 0$
Range: All Real Numbers
x-intercept: $(1, 0)$
Contains the point $(e, 1)$
Vertical Asymptote: $x = 0$
The function is increasing.

Properties of Natural Logarithms

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$
5. If $\cancel{\ln} x = \cancel{\ln} y$, then $x = y$.
6. $\ln(u \bullet v) = \ln u + \ln v$
7. $\ln \frac{u}{v} = \ln u - \ln v$
8. $\ln u^n = n \bullet \ln u$
9. $\ln \sqrt[n]{u} = \frac{1}{n} \bullet \ln u$

$\ln u^{\frac{1}{n}} = \frac{1}{n} \ln u$

Directions: Rewrite each expression in exponential form.

1. $\ln 4 = 1.386$

$\boxed{e^{1.386} = 4}$

2. $\ln 1 = 0$

$\boxed{e^0 = 1}$

Directions: Solve for *x* using the properties of logarithms.

3. $\ln e^7 = x$

$\boxed{7 = x}$

4. $\cancel{\ln} 1 = \cancel{\ln} x$

$\boxed{1 = x}$

Directions: Use the properties to expand each natural logarithm.

5. $\ln\sqrt{\frac{x^2 y}{z}} = \ln\left(\frac{x^2 y}{z}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\ln\frac{x^2 y}{z}$

$= \frac{1}{2}\left[\ln x^2 + \ln y - \ln z\right]$

$= \frac{1}{2}\left[2\ln x + \ln y - \ln z\right]$

$= \boxed{\ln x + \frac{1}{2}\ln y - \frac{1}{2}\ln z}$

6. $\ln\frac{x^3}{yz^4} = \ln x^3 - \ln y - \ln z^4$

$= \boxed{3\ln x - \ln y - 4\ln z}$

7. $\ln\sqrt{x\left(x^2-2\right)} = \ln\left(x\left(x^2-2\right)\right)^{\frac{1}{2}}$

$= \frac{1}{2}\ln x\left(x^2-2\right)$

$= \frac{1}{2}\left[\ln x + \ln\left(x^2-2\right)\right]$

$= \boxed{\frac{1}{2}\ln x + \frac{1}{2}\ln\left(x^2-2\right)}$

Directions: Write each expression as a single logarithm.

8. $\frac{1}{2}[\ln(x+2) - 3\ln x] = \frac{1}{2}\ln(x+2) - \frac{3}{2}\ln x$

$= \ln(x+2)^{\frac{1}{2}} - \ln x^{3/2}$

$= \ln\sqrt{x+2} - \ln\sqrt{x^3}$

$= \boxed{\ln \frac{\sqrt{x+2}}{\sqrt{x^3}}}$

9. $\frac{3}{2}\ln 4x^2 - \frac{2}{3}\ln x^{15}$

$= \ln (4x^2)^{\frac{3}{2}} - \ln (x^{15})^{\frac{2}{3}}$

$= \ln 4^{3/2} \cdot (x^2)^{\frac{3}{2}} - \ln x^{10}$

$= \ln 8 \cdot x^3 - \ln x^{10}$

$= \ln \frac{8x^3}{x^{10}}$

$= \boxed{\ln \frac{8}{x^7}}$

Directions: Solve for *x* in each exponential equation. Round your answer to three decimal places.

10. $\frac{500e^{-x}}{500} = \frac{300}{500}$

$e^{-x} = .6$

$\ln e^{-x} = \ln .6$

$\frac{-x}{-1} = \frac{\ln .6}{-1}$

$x = \frac{-.511}{-1} = \boxed{.511}$

11. $-3 + 4e^{2x} = 5$

$+3 \qquad +3$

$\frac{4e^{2x}}{4} = \frac{8}{4}$

$e^{2x} = 2$

$\ln e^{2x} = \ln 2$

$\frac{2x}{2} = \frac{\ln 2}{2}$

$x = \frac{.693}{2}$

$\boxed{x = .347}$

Directions: Solve for *x* in each logarithmic equation. Round your answer to three decimal places.

12. $\ln x - \ln 5 = \ln 3$

$\ln \frac{x}{5} = \ln 3$

13. $\ln(2x+5) = 10$

e

$\cancel{\ln} \frac{x}{5} = \cancel{\ln} 3$

$\frac{x}{5} = \frac{3}{1}$

$\boxed{x = 15}$

$e^{10} = 2x + \cancel{5}$

$-5 \qquad -\cancel{5}$

$\frac{e^{10} - 5}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$x = \frac{e^{10} - 5}{2} = \frac{22{,}021.466}{2} = \boxed{11{,}010.733}$

14. $\ln \sqrt{x+3} = 3$

e

$(e^3)^2 = (\sqrt{x+3})^2$

$e^6 = x + 3$

$-3 \qquad -3$

$x = e^6 - 3$

$\boxed{x = 400.429}$

15. $\ln(y+6) - \ln y = \ln(y+2)$

$\cancel{\ln} \frac{y+6}{y} = \cancel{\ln}\ y+2$

$\frac{y+6}{y} = \frac{y+2}{1}$

$y + 6 = y(y+2)$

$y + 6 = y^2 + 2y$

$-y - 6 \qquad -y - 6$

$0 = y^2 + y - 6$

$0 = (y+3)(y-2)$

$y + 3 = 0 \qquad y - 2 = 0$

$\cancel{y = -3} \qquad \boxed{y = 2}$