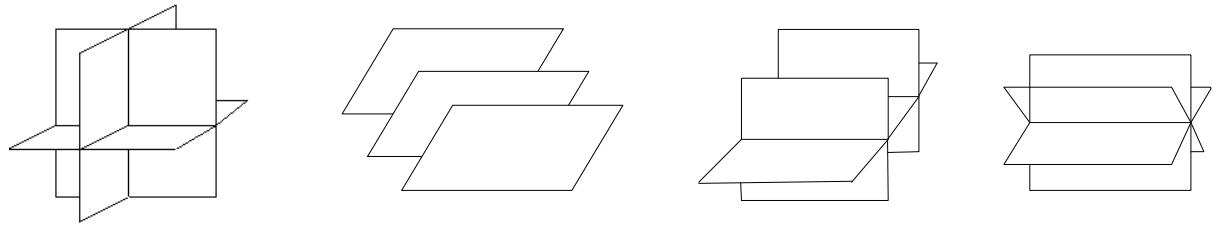
Solving Systems of Equations in Three Variables



One Solution | No Solution | Infinitely Many Solutions

Directions: Solve the system of linear equations.

1. $x-2y+2z=0$ A
   $6x+y+3z=6$ B
   $x+y+z=-4$ C

Use A and B to eliminate y

$$\begin{aligned} x-2y+2z&=0 \\ 2(6x+y+3z&=6) \end{aligned} \rightarrow \begin{aligned} x-2y+2z&=0 \\ 12x+2y+6z&=12 \\ \hline 13x+8z&=12 \end{aligned} \quad D$$

Use A and C to eliminate y

$$\begin{aligned} x-2y+2z&=0 \\ 2(x+y+z&=-4) \end{aligned} \rightarrow \begin{aligned} x-2y+2z&=0 \\ 2x+2y+2z&=-8 \\ \hline 3x+4z&=-8 \end{aligned} \quad E$$

Use D and E to solve for x and z

$$\begin{aligned} 13x+8z&=12 \\ -2(3x+4z&=-8) \end{aligned} \rightarrow \begin{aligned} 13x+8z&=12 \\ -6x-8z&=16 \\ \hline 7x&=28 \\ x&=4 \end{aligned}$$

$$\begin{aligned} 3x+4z&=-8 \\ 3(4)+4z&=-8 \\ 12+4z&=-8 \\ 4z&=-20 \\ z&=-5 \end{aligned}$$

Use A, B or C to solve for y

$$\begin{aligned} x+y+z&=-4 \\ 4+y+-5&=-4 \\ y-1&=-4 \\ y&=-3 \end{aligned}$$

$$\boxed{(4,-3,-5)}$$

2. $2x + y = 2$ A
$y + z = 3$ B
$4x - z = 0$ C

Use B and C to eliminate z

$$\begin{array}{r} y + z = 3 \\ \underline{4x \quad - z = 0} \\ 4x + y = 3 \end{array}$$ D

Use A and D to solve for x and y

$$\begin{array}{r} 2x + y = 2 \\ -1(4x + y = 3) \end{array} \longrightarrow \begin{array}{r} 2x + y = 2 \\ \underline{-4x - y = -3} \\ -2x = -1 \\ x = \frac{1}{2} \end{array}$$

$2x + y = 2$
$2\left(\frac{1}{2}\right) + y = 2$
$1 + y = 2$
$y = 1$

Use B or C to solve for z

$4x - z = 0$
$4\left(\frac{1}{2}\right) - z = 0$
$2 - z = 0$
$-z = -2$
$z = 2$

$$\boxed{\left(\frac{1}{2}, 1, 2\right)}$$

3. $x-5y+4z=8$ A
$3x+y-2z=7$ B
$-9x-3y+6z=5$ C

Use A and B to eliminate z

$$\begin{aligned} x-5y+4z&=8 \\ 2(3x+y-2z&=7) \end{aligned} \longrightarrow \begin{aligned} x-5y+4z&=8 \\ 6x+2y-4z&=14 \\ \hline 7x-3y&=22 \quad D \end{aligned}$$

Use B and C to eliminate z

$$\begin{aligned} 3(3x+y-2z&=7) \\ -9x-3y+6z&=5 \end{aligned} \longrightarrow \begin{aligned} 9x+3y-6z&=21 \\ -9x-3y+6z&=5 \\ \hline 0&=26 \end{aligned}$$

$$\boxed{\begin{array}{l} \text{No solution} \\ \text{INCONSISTENT} \end{array}}$$

4. $4x-6y+8z=4$ A
$5x+y-2z=4$ B
$6x-9y+12z=6$ C

Use A and B to eliminate z

$$\begin{array}{l} 4x-6y+8z=4 \\ 4(5x+y-2z=4) \end{array} \rightarrow \begin{array}{l} 4x-6y+8z=4 \\ 20x+4y-8z=16 \\ \hline 24x-2y=20 \quad D \end{array}$$

Use B and C to eliminate z

$$\begin{array}{l} 6(5x+y-2z=4) \\ 6x-9y+12z=6 \end{array} \rightarrow \begin{array}{l} 30x+6y-12z=24 \\ 6x-9y+12z=6 \\ \hline 36x-3y=30 \quad E \end{array}$$

Use D and E to solve for x and y

$$\begin{array}{l} 3(24x-2y=20) \\ -2(36x-3y=30) \end{array} \rightarrow \begin{array}{l} 72x-6y=60 \\ -72x+6y=-60 \\ \hline 0=0 \end{array}$$

No unique solution

Let $x=a$. Solve for y in D.

$24x-2y=20$

$24a-2y=20$

$-2y=-24a+20$

$y=\dfrac{-24a+20}{-2}$

$y=12a-10$

Solve for z in A, B or C.

$5x+y-2z=4$

$5a+12a-10-2z=4$

$17a-10-2z=4$

$-2z=-17a+14$

$z=\dfrac{-17a+14}{-2}$

$z=\dfrac{17a-14}{2}$

$$\boxed{\left(a,\ 12a-10,\ \frac{17a-14}{2}\right)}$$

5. $5x-2y+z=4$ A

$-3x+4y-z=2$ B

$6x-8y+2z=-4$ C

Use A and B to eliminate z

$$\begin{array}{r} 5x-2y+z=4 \\ -3x+4y-z=2 \\ \hline 2x+2y=6 \\ x+y=3 \end{array}$$ D

Use B and C to eliminate z

$$2(-3x+4y-z=2) \rightarrow \begin{array}{r} -6x+8y-2z=4 \\ 6x-8y+2z=-4 \\ \hline 0=0 \end{array}$$

NO unique solution

Let $x=a$. Solve for y in D.

$x+y=3$

$a+y=3$

$y=3-a$

Solve for z in A, B or C.

$5x-2y+z=4$

$5a-2(3-a)+z=4$

$5a-6+2a+z=4$

$7a-6+z=4$

$7a+z=10$

$z=10-7a$

$$\boxed{(a,\ 3-a,\ 10-7a)}$$