

Sequences

1. Write the first five terms of the sequence whose n^{th} term is shown.

a) $a_n = 5n - 2$

$$a_1 = 5(1) - 2 = 3$$

$$a_2 = 5(2) - 2 = 8$$

$$a_3 = 5(3) - 2 = 13$$

$$a_4 = 5(4) - 2 = 18$$

$$a_5 = 5(5) - 2 = 23$$

$$a_1 = 3$$

$$a_2 = 8$$

$$a_3 = 13$$

$$a_4 = 18$$

$$a_5 = 23$$

b) $a_n = \frac{n+2}{n^2}$

$$a_1 = \frac{1+2}{1^2} = \frac{3}{1} = 3$$

$$a_2 = \frac{2+2}{2^2} = \frac{4}{4} = 1$$

$$a_3 = \frac{3+2}{3^2} = \frac{5}{9}$$

$$a_4 = \frac{4+2}{4^2} = \frac{6}{16} = \frac{3}{8}$$

$$a_5 = \frac{5+2}{5^2} = \frac{7}{25}$$

$$a_1 = 3$$

$$a_2 = 1$$

$$a_3 = \frac{5}{9}$$

$$a_4 = \frac{3}{8}$$

$$a_5 = \frac{7}{25}$$

c) $a_n = (-1)^{n+2}$

$$a_1 = (-1)^{1+2} = (-1)^3 = -1$$

$$a_2 = (-1)^{2+2} = (-1)^4 = 1$$

$$a_3 = (-1)^{3+2} = (-1)^5 = -1$$

$$\begin{array}{l} a_1 = -1 \\ a_2 = 1 \\ a_3 = -1 \\ a_4 = 1 \\ a_5 = -1 \end{array}$$

2. Find the indicated term of the sequence whose n^{th} term is shown.

a) $a_n = \frac{n}{2} - 5$, $a_{12} =$

$$a_{12} = \frac{12}{2} - 5 = 6 - 5 = 1$$

$$a_{12} = 1$$

b) $a_n = (-1)^n (2n-1)$, $a_{25} =$

$$a_{25} = (-1)^{25} (2 \cdot 25 - 1) = (-1)(49) = -49$$

$$a_{25} = -49$$

3. Write the first five terms of the sequence defined recursively.

a) $a_1 = 10$, $a_{k+1} = a_k + 2$

$$\begin{aligned} a_1 &= 10 \\ a_2 &= 12 \\ a_3 &= 14 \\ a_4 &= 14 + 2 = 16 \\ a_5 &= 16 + 2 = 18 \end{aligned}$$

$$a_{k+1} = a_k + 2$$

$K=1$

$$\begin{aligned} a_{1+1} &= a_1 + 2 \\ a_2 &= a_1 + 2 \\ a_2 &= 10 + 2 \\ a_2 &= 12 \end{aligned}$$

$K=2$

$$\begin{aligned} a_{2+1} &= a_2 + 2 \\ a_3 &= a_2 + 2 \\ a_3 &= 12 + 2 \\ a_3 &= 14 \end{aligned}$$

b) $a_1 = 3$, $a_{k+1} = 2a_k - 1$

$$\begin{aligned} a_1 &= 3 \\ a_2 &= 5 \\ a_3 &= 2(5) - 1 = 9 \\ a_4 &= 2(9) - 1 = 17 \\ a_5 &= 2(17) - 1 = 33 \end{aligned}$$

$$a_{k+1} = 2a_k - 1$$

$K=1$

$$\begin{aligned} a_{1+1} &= 2a_1 - 1 \\ a_2 &= 2(3) - 1 \\ &= 6 - 1 \\ &= 5 \end{aligned}$$

4. Write an expression for the n^{th} term of the sequence.

a) 2, 4, 8, 16, ...

$$a_n = 2^n$$

| | | | |
|-------------------|-------|-------|-------|
| a_0 | a_1 | a_2 | a_3 |
| 2 | 4 | 8 | 16 |
| $\rightarrow 2^0$ | 2^1 | 2^2 | 2^3 |
| 2^n | | | |

Check

$$n=5$$

$$a_5 = 2^5 = 32 \checkmark$$

b) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$

| | | | |
|-------------------|-------------------|-------------------|-------------------|
| a_0 | a_1 | a_2 | a_3 |
| $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | $\frac{5}{6}$ |
| $\frac{0+1}{0+2}$ | $\frac{1+1}{1+2}$ | $\frac{2+1}{2+2}$ | $\frac{3+1}{3+2}$ |

$$\frac{n+1}{n+2}$$

$$a_n = \frac{n+1}{n+2}$$

Check

$$n=5$$

$$a_5 = \frac{5+1}{5+2} = \frac{6}{7} \checkmark$$

e) $-1, \frac{1}{2}, -\frac{1}{6}, \frac{1}{24}, -\frac{1}{120}, \dots$

| | | | | | |
|----------------------|----------------------|----------------------|----------------------|----------------------|-----------------|
| a_1 | a_2 | a_3 | a_4 | a_5 | |
| $-\frac{1}{1}$ | $\frac{1}{2}$ | $-\frac{1}{6}$ | $\frac{1}{24}$ | $-\frac{1}{120}$ | $\frac{1}{720}$ |
| $\frac{1(-1)^0}{1!}$ | $\frac{1(-1)^1}{2!}$ | $\frac{1(-1)^2}{3!}$ | $\frac{1(-1)^3}{4!}$ | $\frac{1(-1)^4}{5!}$ | |

$$\frac{1 \cdot (-1)^n}{n!}$$

$$a_n = \frac{(-1)^n}{n!}$$

check
 $n=6$

$$a_6 = \frac{(-1)^6}{6!} = \frac{1}{720} \checkmark$$

Sigma notation

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$$

5. Evaluate each series.

a) $\sum_{i=1}^4 4i+2 = 6 + 10 + 14 + 18 = \boxed{48}$

| | |
|-------|-------------|
| $i=1$ | $4(1)+2=6$ |
| $i=2$ | $4(2)+2=10$ |
| $i=3$ | $4(3)+2=14$ |
| $i=4$ | $4(4)+2=18$ |

$$b) \sum_{i=1}^5 \frac{i^2+3}{i} = 4 + \frac{7}{2} + 4 + \frac{19}{4} + \frac{28}{5} = \boxed{21.85}$$

$$i=1 \quad \frac{1^2+3}{1} = 4$$

$$i=2 \quad \frac{2^2+3}{2} = \frac{7}{2}$$

$$i=3 \quad \frac{3^2+3}{3} = \frac{12}{3} = 4$$

$$i=4 \quad \frac{4^2+3}{4} = \frac{19}{4}$$

$$i=5 \quad \frac{5^2+3}{5} = \frac{28}{5}$$

6. Use sigma notation to write the sum.

a) $3 - 9 + 27 - 81 + 243 - 729$

$$\begin{array}{cccccc}
 i=1 & i=2 & i=3 & i=4 & i=5 & i=6 \\
 3 & -9 & +27 & -81 & +243 & -729 \\
 3^1 & 3^2 & 3^3 & 3^4 & 3^5 & 3^6
 \end{array}$$

$$\boxed{\sum_{i=1}^6 (3^i)(-1)^{i+1}}$$

$$b) \left[1 - \left(\frac{1}{8} \right)^2 \right] + \left[1 - \left(\frac{2}{8} \right)^2 \right] + \dots + \left[1 - \left(\frac{8}{8} \right)^2 \right]$$

$$\begin{aligned} & \overset{i=1}{\left[1 - \left(\frac{1}{8} \right)^2 \right]} + \overset{i=2}{\left[1 - \left(\frac{2}{8} \right)^2 \right]} + \overset{i=3}{\left[1 - \left(\frac{3}{8} \right)^2 \right]} + \overset{i=4}{\left[1 - \left(\frac{4}{8} \right)^2 \right]} + \overset{i=5}{\left[1 - \left(\frac{5}{8} \right)^2 \right]} + \overset{i=6}{\left[1 - \left(\frac{6}{8} \right)^2 \right]} \\ & + \overset{i=7}{\left[1 - \left(\frac{7}{8} \right)^2 \right]} + \overset{i=8}{\left[1 - \left(\frac{8}{8} \right)^2 \right]} \end{aligned}$$

$$\sum_{i=1}^8 \left[1 - \left(\frac{i}{8} \right)^2 \right]$$

$$c) \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \frac{1}{4 \cdot 6} + \dots + \frac{1}{8 \cdot 10}$$

$$\begin{aligned} & \overset{i=1}{\frac{1}{\underline{1 \cdot 3}}} + \overset{i=2}{\frac{1}{\underline{2 \cdot 4}}} + \overset{i=3}{\frac{1}{\underline{3 \cdot 5}}} + \overset{i=4}{\frac{1}{\underline{4 \cdot 6}}} + \overset{i=5}{\frac{1}{\underline{5 \cdot 7}}} + \overset{i=6}{\frac{1}{\underline{6 \cdot 8}}} + \overset{i=7}{\frac{1}{\underline{7 \cdot 9}}} + \overset{i=8}{\frac{1}{\underline{8 \cdot 10}}} \end{aligned}$$

$$\sum_{i=1}^8 \frac{1}{i(i+2)}$$