Geometric Series

2, 4, 8, 16, 32,.....

2 2 2 2

common Ratio = 2

$-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \ldots\ldots$

$-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$

common ratio $= -\frac{1}{2}$

Find the n^{th} term: $a_n = a_1 \bullet r^{n-1}$

a_1: 1^{st} term
r: common ratio
n: term

Sum of a finite geometric sequence with n terms:

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Sum of an infinite geometric sequence:

$$S = \frac{a_1}{1-r} \quad \text{if } |r| < 1$$

1. Write the first 5 terms of each geometric sequence.

a) $a_1 = 6$

$r = -\frac{1}{4}$

$a_1 = 6$

$a_2 = 6 \cdot -\frac{1}{4} = -\frac{6 \div 2}{4 \div 2} = -\frac{3}{2}$

$a_3 = -\frac{3}{2} \cdot -\frac{1}{4} = \frac{3}{8}$

$a_4 = \frac{3}{8} \cdot -\frac{1}{4} = \frac{-3}{32}$

$a_5 = -\frac{3}{32} \cdot -\frac{1}{4} = \frac{3}{128}$

$\{6, -3/2, 3/8, -3/32, 3/128\}$

b) $a_1 = 6$

$a_{k+1} = 2a_k + 1$

$a_{k+1} = 2a_k + 1$

$K = 1$

$a_{1+1} = 2a_1 + 1$

$a_2 = 2a_1 + 1$

$a_2 = 2(6) + 1 = 12 + 1 = 13$

$K = 2$

$a_{2+1} = 2(a_2) + 1$

$a_3 = 2a_2 + 1$

$a_3 = 2(13) + 1 = 26 + 1 = 27$

$K = 3$

$a_1 = 6$

$a_2 = 13$

$a_3 = 27$

$a_4 = 2(27) + 1 = 55$

$a_5 = 2(55) + 1 = 111$

$\{6, 13, 27, 55, 111\}$

2. Find the 10th term of each geometric sequence.

a) $9, -6, 4, -\frac{8}{3}, \ldots$

$R = \frac{-6}{9} = -\frac{2}{3}$

$R = \frac{4}{-6} = -\frac{2}{3}$

$a_N = a_1 \cdot R^{N-1}$

$a_{10} = ?$

$N = 10$

$a_1 = 9$

$R = -\frac{2}{3}$

$a_{10} = 9 \cdot \left(\frac{-2}{3}\right)^{10-1}$

$a_{10} = 9\left(\frac{-2}{3}\right)^9$

$a_{10} = 9 \cdot \frac{(-2)^9}{(3)^9}$

$a_{10} = \frac{\cancel{9}^{1} \cdot (-512)}{\cancel{19683}_{2187}} = \boxed{\frac{-512}{2187}}$

b) $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, \ldots$

$R = \frac{2}{1} = 2$

$a_N = a_1 \cdot R^{N-1}$

$a_{10} = ?$

$N = 10$

$a_1 = \frac{1}{8}$

$R = 2$

$a_{10} = \frac{1}{8}(2)^{10-1}$

$a_{10} = \frac{1}{8}(2)^9$

$a_{10} = \frac{1}{\cancel{8}_1} \cdot \overset{64}{\cancel{512}}$

$a_{10} = \boxed{64}$

3. Write an expression for the n^{th} term of the geometric sequence.

a) $a_1 = 60$

$r = \frac{1}{3}$

$n = 10$

$a_{10} = ?$

$a_N = a_1 R^{N-1}$

$a_{10} = 60 \cdot \left(\frac{1}{3}\right)^{10-1}$

$a_{10} = 60 \cdot \left(\frac{1}{3}\right)^9$

$a_{10} = 60 \cdot \frac{1^9}{3^9}$

$a_{10} = \overset{20}{\cancel{60}} \cdot \frac{1}{\underset{6561}{\cancel{19683}}}$

$\boxed{a_{10} = \frac{20}{6561}}$

b) $a_3 = \frac{4}{3}$

$r = -\frac{1}{9}$

$n = 6$

$a_6 = ?$

$a_N = a_1 R^{N-1}$

$a_6 = 108 \cdot \left(-\frac{1}{9}\right)^{6-1}$

$a_6 = 108 \cdot \left(-\frac{1}{9}\right)^5$

$a_6 = \overset{4}{\cancel{108}} \cdot \frac{-1}{\underset{2187}{\cancel{59049}}} = \boxed{\frac{-4}{2187}}$

To find a_1, use $a_N = a_1 R^{N-1}$

$R = -\frac{1}{9}$

$a_N = a_3 = \frac{4}{3}$

$N = 3$

$a_3 = a_1\left(-\frac{1}{9}\right)^{3-1}$

$\frac{4}{3} = a_1\left(-\frac{1}{9}\right)^2$

$\overset{27}{\cancel{81}} \cdot \frac{4}{\cancel{3}} = a_1 \cdot \frac{1}{\cancel{81}} \cdot \cancel{81}$

$a_1 = 108$

4. Find the sum of the finite geometric sequence.

$$\sum_{i=1}^{8} 2^{n-1}$$

$a_1 = 1$

$a_2 = 2^{2-1} = 2$

$$S_N = a_1\left(\frac{1-R^N}{1-R}\right)$$

$a_1 = 2^{1-1} = 2^0 = 1$

$R = 2$

$N = 8 - 1 + 1 = 8$

$$S_8 = 1\left(\frac{1-2^8}{1-2}\right)$$

$$= \frac{1-256}{-1}$$

$$= \frac{-255}{-1}$$

$$= \boxed{255}$$

5. Use summation notation to express the sum.

a) $4 + 12 + 36 + \cdots + 8,748$

$3 \quad 3 \qquad a_N$

$$\sum_{N=1}^{8} a_1 R^{N-1}$$

$a_N = a_1 R^{N-1}$

$\frac{8748}{4} = \frac{4}{4} \cdot 3^{N-1}$

$2187 = 3^{N-1}$

$3^1 = 1 \quad 3^2 = 9 \quad 3^3 = 27 \quad 3^4 = 81 \quad 3^5 = 243 \quad 3^6 = 729 \quad 3^7 = 2187$

$a_1 = 4$

$R = 3$

$N = 8$

$$\boxed{\sum_{N=1}^{8} 4 \cdot 3^{N-1}}$$

b) $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \cdots + \frac{1}{2,048}$

$a_1 \, a_2 \, a_3 \, a_4 \qquad a_N$

$R: \frac{-\frac{1}{2}}{2} = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4}$

$$\sum_{N=1}^{7} a_1 (R)^{N-1} = \boxed{\sum_{N=1}^{7} 2\left(-\frac{1}{4}\right)^{N-1}}$$

$a_1 = 2$

$R = -\frac{1}{4}$

$N = 7$

$a_N = a_1 R^{N-1}$

$\frac{1}{2} \cdot \frac{1}{2048} = \frac{2}{2}\left(-\frac{1}{4}\right)^{N-1}$

$\frac{1}{4096} = \left(-\frac{1}{4}\right)^{N-1}$

$N=7$

$4^1=4$ $4^4=256$

$4^2=16$ $4^5=1024$

$4^3=64$ $4^6=4096$

6. Find the sum of the infinite geometric series.

a) $\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n$

$S_N = \frac{a_1}{1-R}$ $\underbrace{|R|<1}$

$R=\frac{1}{10}$

$\left|\frac{1}{10}\right|<1$

$\frac{1}{10}<1$ ✓

$a_1=\left(\frac{1}{10}\right)^0=1$

$R=\frac{1}{10}$

$S=\frac{1}{10\cdot\frac{1}{10\cdot 1}-\frac{1}{10}}=\frac{1}{\frac{10}{10}-\frac{1}{10}}=\frac{1}{\frac{9}{10}}=\boxed{\frac{10}{9}}$

b) $8+6+\frac{9}{2}+\frac{27}{8}+\cdots$

$S_N=\frac{a_1}{1-R}$ $\underbrace{|R|<1}$

$R: \frac{6}{8}=\frac{3}{4}$

$\left|\frac{3}{4}\right|<1$

$\frac{3}{4}<1$ ✓

$a_1=8$

$R=\frac{3}{4}$

$S=\frac{8}{4\cdot\frac{1}{4\cdot 1}-\frac{3}{4}}=\frac{8}{\frac{4}{4}-\frac{3}{4}}=\frac{8}{\frac{1}{4}}$

$S=8\cdot\frac{4}{1}=\boxed{32}$