

Evaluating the Limit of a Function at a Point Algebraically

Basic Limit Theorems

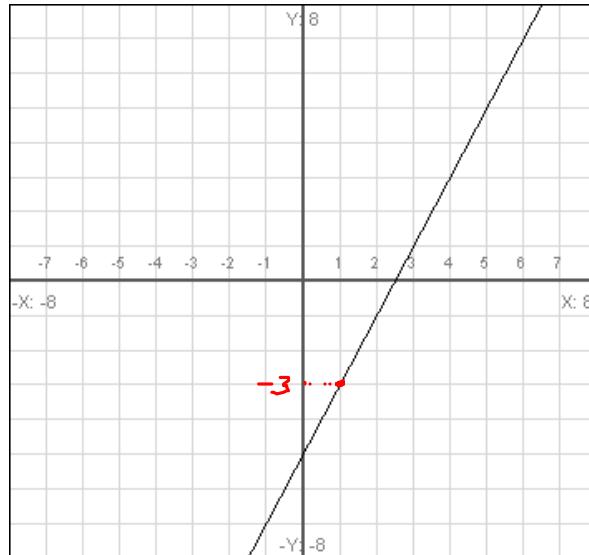
Theorem 1:

$$\lim_{x \rightarrow a} mx + b = ma + b$$

Example 1: Evaluate the limit.

$$\lim_{x \rightarrow 1} 2x - 5 = 2(1) - 5$$

$$2 - 5 = \boxed{-3}$$



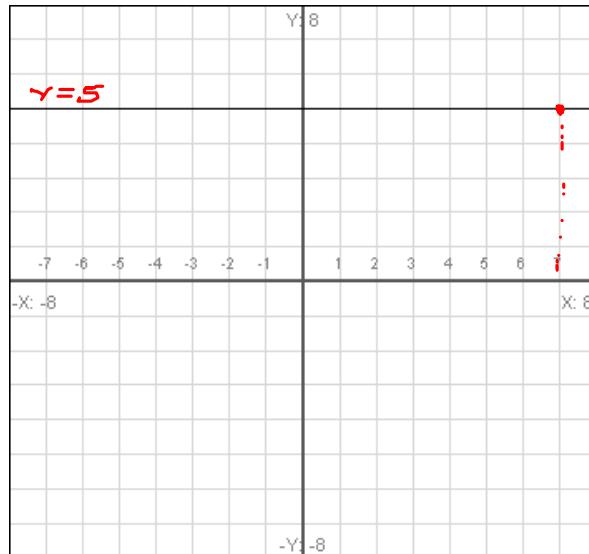
Theorem 2:

$$\lim_{x \rightarrow a} c = c$$

Example 2: Evaluate the limit.

$$\lim_{x \rightarrow 7} 5 = \boxed{5}$$

$y=5$
horizontal line



Theorem 3:

$$\lim_{x \rightarrow a} x^n = a^n$$

Example 3: Evaluate each limit.

a) $\lim_{x \rightarrow 2} x^3 = (2)^3 = \boxed{8}$

b) $\lim_{x \rightarrow 8} x^{\frac{5}{3}} = (8)^{\frac{5}{3}} = \frac{1}{8^{\frac{5}{3}}} = \frac{1}{(\sqrt[3]{8})^5} = \frac{1}{2^5} = \boxed{\frac{1}{32}}$

Theorem 4:

$$\lim_{x \rightarrow a} cx^n = c \cdot \lim_{x \rightarrow a} x^n = ca^n$$

Example 4: Evaluate the limit.

a) $\lim_{x \rightarrow 9} 4\sqrt{x} = 4 \lim_{x \rightarrow 9} \sqrt{x} = 4\sqrt{9} = 4(3) = \boxed{12}$

Theorem 5:

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = f(a) \pm g(a)$$

Example 5: Evaluate each limit.

$$\lim_{x \rightarrow 2} x^3 + 5x + 7 = \lim_{x \rightarrow 2} x^3 + \lim_{x \rightarrow 2} 5x + \lim_{x \rightarrow 2} 7 = (2)^3 + 5(2) + 7 = 8 + 10 + 7 = \boxed{25}$$

Theorem 6:

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) = f(a) \cdot g(a)$$

Example 6: Evaluate the limit.

$$\begin{aligned}\lim_{x \rightarrow 1} (3x+5)(x^3 - 2) &= \lim_{x \rightarrow 1} (3x+5) \cdot \lim_{x \rightarrow 1} (x^3 - 2) = [3(1)+5] \cdot [(1)^3 - 2] \\ &= 8 \cdot (-1) \\ &= \boxed{-8}\end{aligned}$$

Theorem 7:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{f(a)}{g(a)} \text{ provided } g(a) \neq 0$$

Example 7: Evaluate the limit.

$$\lim_{x \rightarrow 3} \frac{x+1}{x+5} = \frac{\lim_{x \rightarrow 3} x+1}{\lim_{x \rightarrow 3} x+5} = \frac{3+1}{3+5} = \frac{4}{8} = \boxed{\frac{1}{2}}$$