Algebraic Techniques to Evaluate Limits When the Denominator Equals Zero

Step 1: Substitute in the value into the limit.
Step 2: Apply an algebraic technique.
Step 3: Substitute the value into the simplified limit.

Directions: Evaluate each limit.

1) $\lim_{x\to 2}\frac{x^2-3x+2}{x^2+x-6} = \frac{2^2-3(2)+2}{2^2+2-6} = \frac{4-6+2}{4+2-6} = \frac{0}{0}$

Algebraic Technique: Factoring

$$\lim_{x\to 2}\frac{\cancel{(x-2)}(x-1)}{(x+3)\cancel{(x-2)}} = \frac{x-1}{x+3} = \frac{2-1}{2+3} = \boxed{\frac{1}{5}}$$

2) $\lim_{x\to 1}\frac{x^3-1}{x-1} = \frac{1^3-1}{1-1} = \frac{1-1}{1-1} = \frac{0}{0}$

Algebraic Technique: Factoring

$$A^3-B^3=(A-B)(A^2+AB+B^2)$$
$$x^3-1=(x-1)(x^2+x+1)$$
x x x   1 1 1
A = x   B = 1

$$\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = x^2+x+1 = 1^2+1+1$$
$$= 1+1+1$$
$$= \boxed{3}$$

3) $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x} = \frac{\sqrt{0+1}-1}{0} = \frac{\sqrt{1}-1}{0} = \frac{1-1}{0} = \frac{0}{0}$

Algebraic Technique: Multiply the Fraction by the Conjugate of the Numerator

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \frac{(x+1)-1}{x(\sqrt{x+1}+1)} = \frac{\cancel{x}\ 1}{\cancel{x}(\sqrt{x+1}+1)} = \frac{1}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{0+1}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

4) $\lim\limits_{x\to 7}\dfrac{5-\sqrt{4+3x}}{x-7} = \dfrac{5-\sqrt{4+3\cdot 7}}{7-7} = \dfrac{5-\sqrt{25}}{7-7} = \dfrac{0}{0}$

Algebraic Technique: Multiply the Fraction by the Conjugate of the Numerator

$$\lim_{x\to 7} \frac{5-\sqrt{4+3x}}{x-7}\cdot\frac{5+\sqrt{4+3x}}{5+\sqrt{4+3x}} = \frac{25-(4+3x)}{(x-7)(5+\sqrt{4+3x})} = \frac{25-4-3x}{(x-7)(5+\sqrt{4+3x})}$$

$$\lim_{x\to 7} \frac{21-3x}{(x-7)(5+\sqrt{4+3x})} = \frac{3(\overset{-1}{\cancel{7-x}})}{(\cancel{x-7})(5+\sqrt{4+3x})} = \frac{-3}{5+\sqrt{4+3x}} = \frac{-3}{5+\sqrt{4+3\cdot 7}}$$

$$= \frac{-3}{5+\sqrt{25}} = \boxed{\frac{-3}{10}}$$

5) $\lim\limits_{x\to 0}\dfrac{x^2+3x-1}{x}+\dfrac{1}{x} = \dfrac{0^2+3(0)-1}{0}+\dfrac{1}{0} = \dfrac{-1}{0}+\dfrac{1}{0} = \dfrac{0}{0}$

Algebraic Technique: Combine Fractions

$$\lim_{x\to 0} \frac{x^2+3x\cancel{-1}\cancel{+1}}{x} = \frac{x^2+3x}{x} = \frac{\cancel{x}(x+3)}{\cancel{x}} = x+3 = 0+3 = \boxed{3}$$

6) $\lim_{h \to 0} \frac{1}{h}\left(\frac{6+h}{3+2h} - 2\right) =$ $\frac{1}{0}\left(\frac{6+0}{3+2(0)} - 2\right) = \frac{1}{0}(2-2) = \frac{0}{0}$

Algebraic Technique: Combine Fractions

$\lim_{h \to 0} \frac{1}{h}\left(\frac{6+h}{3+2h} - \frac{2 \cdot 3+2h}{1 \cdot 3+2h}\right) = \frac{1}{h}\left(\frac{6+h}{3+2h} - \frac{2(3+2h)}{3+2h}\right) = \frac{1}{h}\left(\frac{\cancel{6}+h-\cancel{6}-4h}{3+2h}\right)$

LCD $= 3+2h$

$\lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{-3\cancel{h}}{3+2h}\right) = \frac{-3}{3+2h} = \frac{-3}{3+2(0)} = \frac{-3}{3} = \boxed{-1}$

7) $\lim_{x \to 3} \frac{|x-3|}{x-3} =$ $\frac{|3-3|}{3-3} = \frac{|0|}{0} = \frac{0}{0}$

Algebraic Technique: One-Sided Limits

$\lim_{x \to 3^+} \frac{|3.1-3|}{3.1-3} = \frac{|.1|}{.1} = \frac{.1}{.1} = 1$

$\lim_{x \to 3^-} \frac{|2.9-3|}{2.9-3} = \frac{|-.1|}{-.1} = \frac{.1}{-.1} = -1$

$\lim_{x \to 3} \frac{|x-3|}{x-3}$ $\boxed{\text{Does not exist}}$