Limits Involving Infinity (Limit Approaches Infinity)

For Rational Functions Only:

Method 1: Divide each term by the highest power in the denominator.

Method 2: Compare the degree of the numerator to the degree of the denominator.

$$\lim_{x\to\pm\infty}\frac{f(x)}{g(x)}=\begin{cases}\text{If degree of } f(x) = \text{degree of } g(x),\ \text{then } \dfrac{\text{leading coefficient}}{\text{leading coefficient}}\\ \text{If degree of } f(x) > \text{degree of } g(x),\ \text{then } \infty\\ \text{If degree of } f(x) < \text{degree of } g(x),\ \text{then } 0\end{cases}$$

Directions: Evaluate each limit.

1) $\lim_{x\to\infty}\frac{2x^{1}-5}{4x^{1}-1}=$ highest power in denominator = x

Method 1: $\lim_{x\to\infty}\dfrac{\frac{2\cancel{x}}{\cancel{x}}-\frac{5}{x}}{\frac{4\cancel{x}}{\cancel{x}}-\frac{1}{x}}=\dfrac{2-\frac{5}{x}\to 0}{4-\frac{1}{x}\to 0}=\dfrac{2-0}{4-0}=\dfrac{2}{4}=\boxed{\dfrac{1}{2}}$

Method 2: degree numerator = x^1 degree denominator = x^1

$\dfrac{2}{4}=\boxed{\dfrac{1}{2}}$

2) $\lim_{x\to\infty}\frac{3x^2-2x+7}{4-5x^2}=$ highest power in denominator = x^2

Method 1: $\lim_{x\to\infty}\dfrac{\frac{3\cancel{x^2}}{\cancel{x^2}}-\frac{2\cancel{x}}{x^{\cancel{2}}}+\frac{7}{x^2}}{\frac{4}{x^2}-\frac{5\cancel{x^2}}{\cancel{x^2}}}=\dfrac{3-\frac{2}{x}\to 0+\frac{7}{x^2}\to 0}{\frac{4}{x^2}\to 0-5}=\dfrac{3-0+0}{0-5}=\boxed{\dfrac{-3}{5}}$

Method 2: degree numerator = x^2 degree denominator = x^2

$\boxed{\dfrac{3}{-5}}$

3) $\lim_{x\to\infty}\frac{4x-3}{\sqrt{9x^2+1}}=$ highest power in denominator x^2

Method 1: $\lim_{x\to\infty}\dfrac{\frac{4\cancel{x}}{\cancel{x}}-\frac{3}{x}}{\sqrt{\frac{9\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}}}=\dfrac{4-\frac{3}{x}\to 0}{\sqrt{9+\frac{1}{x^2}\to 0}}=\dfrac{4-0}{\sqrt{9+0}}=\dfrac{4}{\sqrt{9}}=\boxed{\dfrac{4}{3}}$

Method 2: degree numerator = x^1 degree denominator = $\sqrt{x^2}=x^1$

$$\frac{4}{\sqrt{9}} = \boxed{\frac{4}{3}}$$

4) $\lim_{x\to\infty}\frac{4x^2}{3x^3+x^2-7}=$ highest power in denominator $= x^3$

Method 1: $\lim_{x\to\infty} \frac{\frac{4x^2}{x^3}}{\frac{3x^3}{x^3}+\frac{x^2}{x^3}-\frac{7}{x^3}} = \frac{\frac{4}{x}\to 0}{3+\frac{1}{x}-\frac{7}{x^3}\to 0} = \frac{0}{3+0-0} = \frac{0}{3} = \boxed{0}$

Method 2:

degree numerator $= x^2$ degree denominator $= x^3$

$\boxed{0}$

5) $\lim_{x\to\infty}\frac{10^7x^4}{10^6x^5+7^{10}x^2+1}=$ highest power in denominator $= x^5$

Method 1: $\lim_{x\to\infty} \frac{\frac{10^7x^4}{x^5}}{\frac{10^6x^5}{x^5}+\frac{7^{10}x^2}{x^5}+\frac{1}{x^5}} = \frac{\frac{10^7}{x}\to 0}{10^6+\frac{7^{10}}{x^3}+\frac{1}{x^5}} = \frac{0}{10^6+0+0} = \frac{0}{10^6} = \boxed{0}$

Method 2:

degree numerator $= x^4$ degree denominator $= x^5$

$\boxed{0}$

6) $\lim_{x\to\infty}\frac{x^8}{2-3x^5+7x^6+4x^7}=$ highest power in denominator $= x^7$

Method 1: $\lim_{x\to\infty} \frac{\frac{x^8}{x^7}}{\frac{2}{x^7}-\frac{3x^5}{x^7}+\frac{7x^6}{x^7}+\frac{4x^7}{x^7}} = \frac{x}{\frac{2}{x^7}-\frac{3}{x^2}+\frac{7}{x}+4} = \frac{x}{0+0+0+4} = \frac{x}{4} = \frac{\infty}{4} = \boxed{\infty}$

Method 2:

degree numerator $= x^8$ degree denominator $= x^7$

$\boxed{\infty}$