Continuity

<u>A function is continuous at *x* = *a* if:</u>

$a)\ f(a)$ exists.

$b)\ \lim_{x\to a} f(x)$ exists.

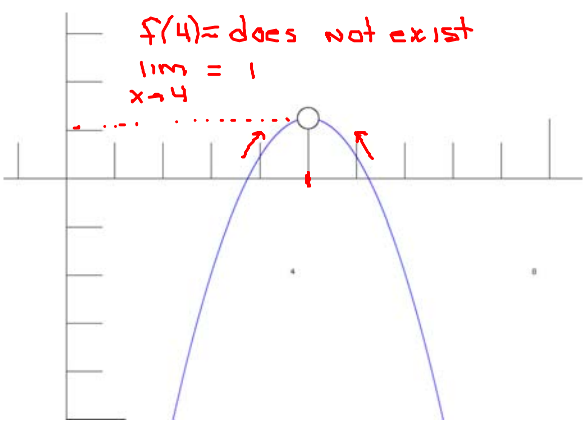
$c)\ \lim_{x\to a} f(x) = f(a)$

<u>A function is not continuous at:</u>

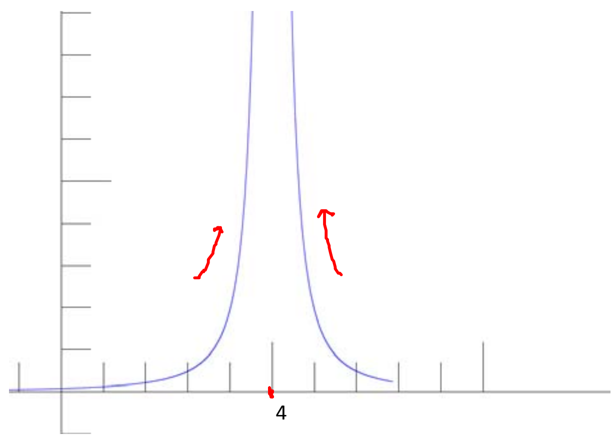
a) Vertical Asymptotes
b) Deleted Points/Holes
c) Breaking Points

<u>Examples of Discontinuous Functions:</u>

Deleted Point/Hole at *x* = 4



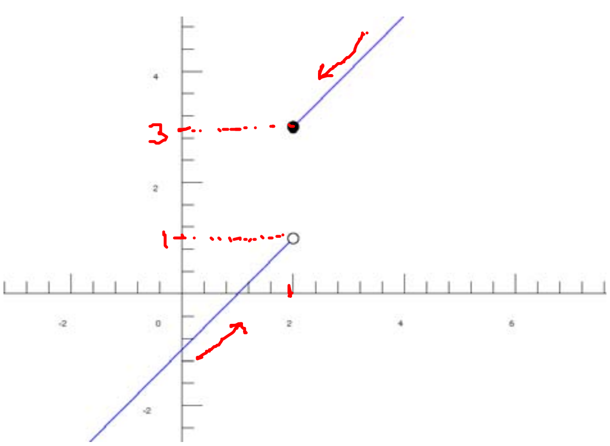
Asymptote at *x* = 4



f(4)= does not exist

$\lim_{x\to 4} = \infty$

Breaking Point at *x* = 2



f(2)= 3

$\lim_{x\to 2^-} = 1$

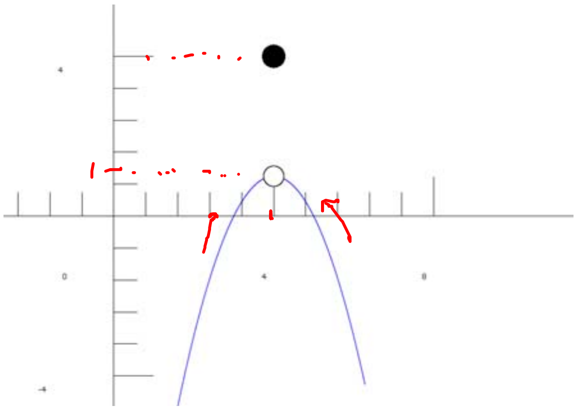
$\lim_{x\to 2^+} = 3$

$\lim_{x\to 2}$ does not exist

Discontinuous at *x* = 4 because

$\lim_{x\to 4} f(x) \neq f(4)$

$\lim_{x \to 4} f(x) \neq f(4)$



$f(4) = 4$

$\lim_{x \to 4} = 1$

1. Determine if the function is continuous at $x = 2$.

$a)\ f(x) = \dfrac{1}{x-2}$

$f(2) = \dfrac{1}{2-2} = \dfrac{1}{0} = \text{undefined}$

$\boxed{\text{discontinuous at } x=2}$

$b)\ f(x) = \begin{cases} 3x^2 - 1 & \text{for } x < 2 \\ 2x + 5 & \text{for } x \geq 2 \end{cases}$

$f(2) = 2(2)+5 = 4+5 = 9$

$\lim_{x \to 2^-} 3(2)^2 - 1 = 3(4) - 1 = 12 - 1 = 11$

$\lim_{x \to 2^+} = 9$

$\boxed{\text{discontinuous at } x=2}$

$c)\ f(x) = \begin{cases} \dfrac{x^2 - 4}{x-2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$

$f(2) = 5$

$\lim_{x \to 2} \dfrac{x^2-4}{x-2} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = x+2 = 2+2 = 4$

$\boxed{\text{discontinuous at } x=2}$

2. Find c if $f(x)$is continuous at $x = 2$.

$$f(x)=\begin{cases} x+3 & \text{for } x \le 2 \\ cx+6 & \text{for } x > 2 \end{cases}$$

$f(2) = 2+3 = 5$

$\lim_{x\to 2^-} = 5$

$\lim_{x\to 2^+} c(2)+6 = 2c+6$

$2c + \cancel{6} = 5$
$\quad \cancel{-6} \quad -6$

$\frac{\cancel{2}c}{\cancel{2}} = -\frac{1}{2}$

$\boxed{c = -1/2}$

3. Find c if $f(x)$ is continuous at $x = 3$.

Difference of 2 cubes

$$f(x)=\begin{cases} \dfrac{x^3-27}{x-3} & \text{if } x \ne 3 \\ c & \text{if } x = 3 \end{cases}$$

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$x^3 - 27 = (x-3)(x^2+3x+9)$

$x \; x \; x \qquad 3 \; 3 \; 3$

$A = x \quad B = 3$

$f(3) = c$

$\lim_{x\to 3} \frac{x^3-27}{x-3}$

$\lim_{x\to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}} = x^2+3x+9 = 3^2+3(3)+9 = 9+9+9 = 27$

$\boxed{c = 27}$

4. Determine the points where the function is discontinuous.

a) $f(x) = x^2 - 2x + 1$

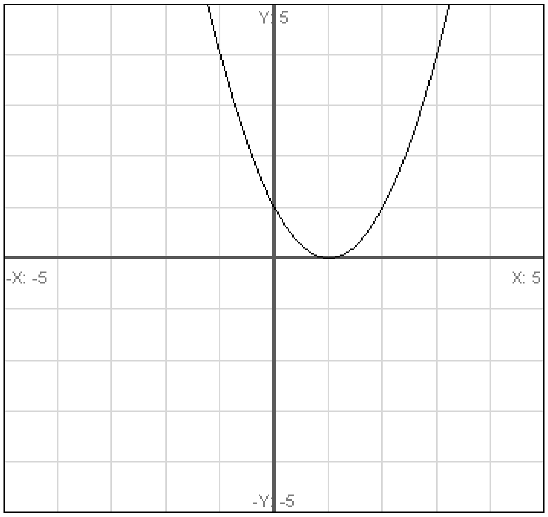
$\boxed{\text{Continuous Everywhere}}$

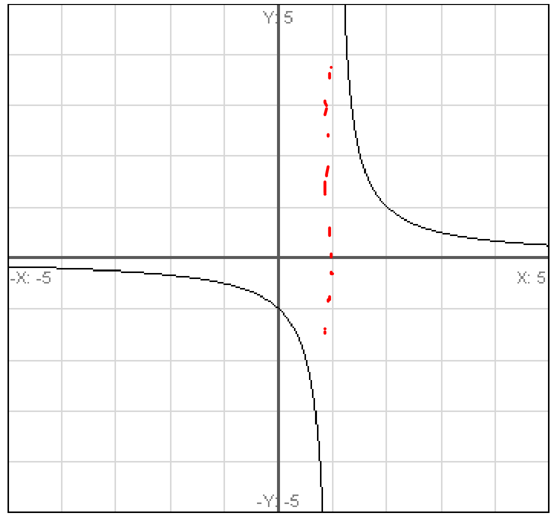
b) $f(x) = \dfrac{1}{x-1}$

$x - 1 \ne 0$

$x \ne 1$

$\boxed{\text{Discontinuous at } x = 1}$



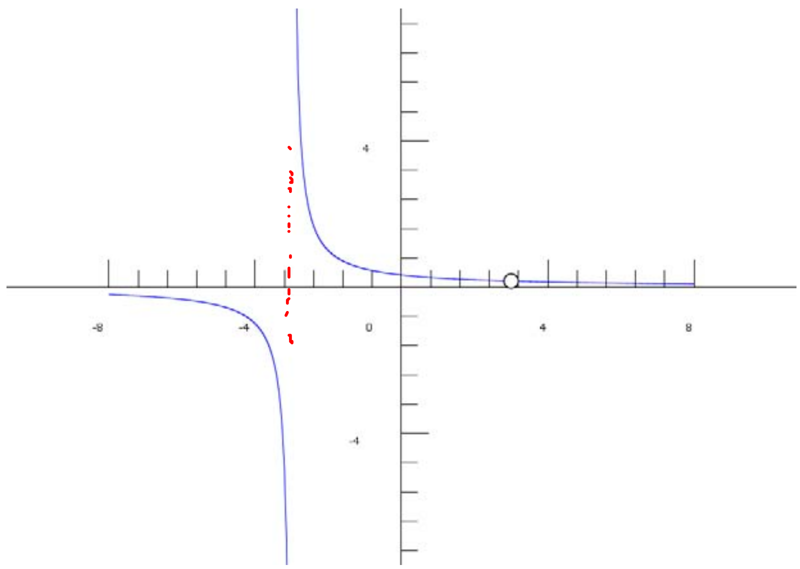


c) $f(x) = \dfrac{x-3}{x^2-9}$

$x^2 - 9 \neq 0$
$+9 \quad +9$
$\sqrt{x^2} \neq \sqrt{9}$

$x \neq \pm 3$

$\boxed{\text{Discontinuous at } x = \pm 3}$



d) $f(x) = \dfrac{x}{x^2+1}$

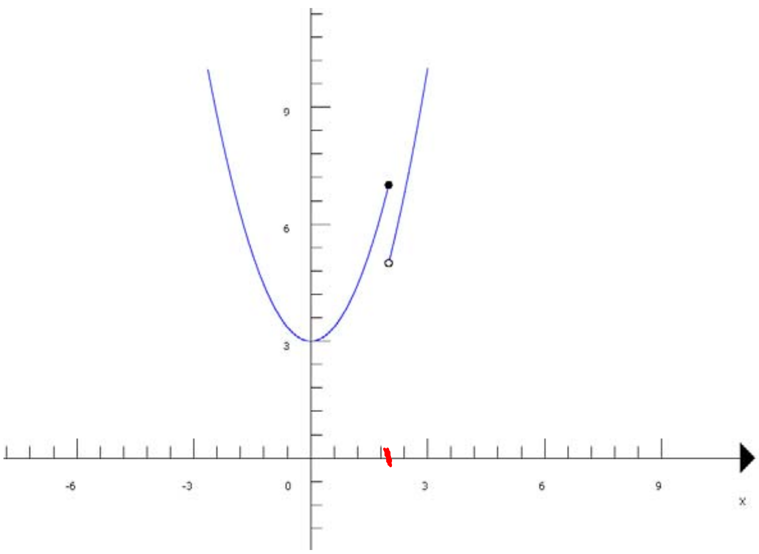
$x^2+1 \neq 0$

$-1 \quad -1$

$\sqrt{x^2} \neq \sqrt{-1}$

$\boxed{\text{continuous everywhere}}$

e) $f(x) = \begin{cases} 3+x^2 & \text{for } x \le 2 \\ x^2+1 & \text{for } x > 2 \end{cases}$



$f(2) = 3+2^2 = 3+4 = 7$

$\lim_{x \to 2^-} = 7$

$\lim_{x \to 2^+} = 2^2+1 = 4+1 = 5$

$\lim_{x \to 2}$ does not exist

$\boxed{\text{discontinuous at } x=2}$