Basic Differentiation Rules

Theorem 1: If $f(x) = c$, then $f'(x) = 0$

Example 1: Find $f'(x)$.

$a)\ f(x) = 2$

$\boxed{f'(x) = 0}$

$b)\ f(x) = \pi$

$\boxed{f'(x) = 0}$

Theorem 2: If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Example 2: Find $f'(x)$.

$a)\ f(x) = x^5 \qquad n = 5$

$f'(x) = 5x^{5-1} = \boxed{5x^4}$

$b)\ f(x) = \sqrt{x} = x^{\frac{1}{2}} \qquad n = \frac{1}{2}$

$f'(x) = \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$

Example 3:

Find the slope of the curve $y = \sqrt[4]{x^3}$ at $x = 16$.

$y = x^{3/4} \qquad n = \frac{3}{4}$

$y' = \frac{3}{4} \cdot x^{\frac{3}{4}-1} = \frac{3}{4} \cdot x^{-\frac{1}{4}} = \frac{3}{4} \cdot \frac{1}{x^{1/4}} = \boxed{\frac{3}{4\sqrt[4]{x}}}$

$y'\Big|_{x=16} = \frac{3}{4\sqrt[4]{16}} = \frac{3}{4 \cdot 2} = \boxed{\frac{3}{8}}$

$2 \times 2 \times 2 \times 2 = 16$

$\sqrt[4]{16} = 2$

Theorem 3: If $f(x) = cx^n$ then $f'(x) = n \cdot cx^{n-1}$

Example 4: Find $f'(x)$.

$a)\ 6x^3 \qquad c = 6 \qquad n = 3$

$f'(x) = 3 \cdot 6x^{3-1} = \boxed{18x^2}$

$b)\ 9\sqrt[3]{x} = 9x^{\frac{1}{3}} \qquad c = 9 \qquad n = \frac{1}{3}$

$f'(x) = \frac{1}{3} \cdot 9x^{\frac{1}{3}-1} = 3x^{-2/3}$

$= 3 \cdot \frac{1}{x^{2/3}} = \boxed{\frac{3}{x^{2/3}}}$

If $f(x) = g(x) \pm h(x)$ then $f'(x) = g'(x) \pm h'(x)$

Theorem 4: If $f(x)=g(x)\pm h(x)$ then $f'(x)=g'(x)\pm h'(x)$

Example 5: Find *f'(x)*.

a) $f(x)=3x^2+2x^1+1$

$f'(x)=2\cdot 3x^{2-1}+1\cdot 2x^{1-1}+0$

$=6x+2\underbrace{x^0}_{1}$

$=\boxed{6x+2}$

b) $f(x)=4x^5-3x^4+2x^3-7x^2+6x-1$

$f'(x)=20x^4-12x^3+6x^2-14x^1+6-0$

$\boxed{=20x^4-12x^3+6x^2-14x+6}$

c) $f(x)=\sqrt{x}(x+5)=x^{\frac{1}{2}}(x^1+5)$

$f(x)=x^{3/2}+5x^{1/2}$

$f'(x)=\frac{3}{2}x^{1/2}+\frac{5}{2}x^{-1/2}$

$=\frac{3}{2}\sqrt{x}+\frac{5}{2}\cdot\frac{1}{\sqrt{x}}$

$=\boxed{\frac{3}{2}\sqrt{x}+\frac{5}{2\sqrt{x}}}$

d) $f(x)=\frac{6}{\sqrt[3]{x}}+\frac{4}{x^2}-7$

$f(x)=6x^{-\frac{1}{3}}+4x^{-2}-7$

$f'(x)=-2x^{-\frac{1}{3}-1}+-8x^{-2-1}-0$

$f'(x)=-2x^{-4/3}-8x^{-3}$

$=\boxed{\frac{-2}{x^{4/3}}-\frac{8}{x^3}}$

e) $f(x)=\frac{6x^4+8x^3}{2x^2}=\frac{6x^4}{2x^2}+\frac{8x^3}{2x^2}=3x^2+4x$

$f'(x)=6x^1+4$

$f'(x)=\boxed{6x+4}$

f) $f(x)=\frac{7x^7}{4}=\frac{7}{4}x^7$

$f'(x)=\frac{7}{4}\cdot 7x^6=\boxed{\frac{49}{4}x^6}$

Theorem 5: If $f(x)=c\cdot g(x)$ then $f'(x)=c\cdot g'(x)$

Example 6: Find *f'(x)*.

$f(x)=3(x^2+7)$ $\quad c=3$

$f'(x)=3(2x^1+0)=3(2x)=\boxed{6x}$