Chain Rule

If $f(x) = u^n$ (where u is a function of x) then $f'(x) = n \bullet u^{n-1} \bullet \frac{du}{dx}$

Directions: Find the derivative of each.

1. $f(x) = (3x-1)^{10}$ $\quad f(x) = (3x-1)^{10}$

$f'(x) = 10(3x-1)^9 \cdot (3) = \boxed{30(3x-1)^9}$

2. $f(x) = (5x^2 - 3x + 1)^4$ $\quad f(x) = (5x^2-3x+1)^4$

$f'(x) = 4(5x^2-3x+1)^3 \cdot (10x-3) = \boxed{4(10x-3)(5x^2-3x+1)^3}$

3. $f(x) = 9\sqrt[3]{4-x^2} = 9(4-x^2)^{\frac{1}{3}}$

$f'(x) = 3(4-x^2)^{-\frac{2}{3}} \cdot (-2x) = \boxed{\frac{-6x}{(4-x^2)^{\frac{2}{3}}}}$

4. $f(x) = \underbrace{5x^2}_{1st}\underbrace{(3x-4)^3}_{2nd}$ $\quad f(x) = 5x^2(3x-4)^3$

$f'(x) = 10x(3x-4)^3 + 5x^2 \cdot 3(3x-4)^2 \cdot (3)$

$= \frac{\overset{2}{\cancel{10}}x(3x-4)^{\overset{1}{\cancel{3}}}}{\cancel{5x(3x-4)^2}} + \frac{\overset{9x}{\cancel{45}}x^2\cancel{(3x-4)^2}}{\cancel{5x(3x-4)^2}}$ $\quad GCF = 5x(3x-4)^2$

$= 5x(3x-4)^2[2(3x-4) + 9x]$

$= 5x(3x-4)^2[6x-8+9x]$

$= \boxed{5x(3x-4)^2[15x-8]}$

5. $f(x)=(3x^2-2)^2(3x-1)^3$ $\quad f(x)=(3x^2-2)^2\cdot(3x-1)^3$

$$f'(x)=2(3x^2-2)^1\cdot(6x)(3x-1)^3+(3x^2-2)^2\cdot 3(3x-1)^2(3)$$

$$f'(x)=\frac{\overset{4x}{\cancel{12}x}\cancel{(3x^2-2)}(3x-1)^{\overset{1}{\cancel{3}}}}{\cancel{3(3x^2-2)(3x-1)^2}}+\frac{\overset{3}{\cancel{9}}(3x^2-2)^{\overset{1}{\cancel{2}}}\cancel{(3x-1)}^2}{\cancel{3(3x^2-2)(3x-1)^2}}$$

GCF $=3(3x^2-2)(3x-1)^2$

$$f'(x)=3(3x^2-2)(3x-1)^2\left[4x(3x-1)+3(3x^2-2)\right]$$
$$=3(3x^2-2)(3x-1)^2(12x^2-4x+3x^2-6)$$
$$\boxed{=3(3x^2-2)(3x-1)^2(15x^2-4x-6)}$$

6. $y=\dfrac{2x+1}{\sqrt{x+1}}$ $\qquad y=\dfrac{2x+1}{(x+1)^{1/2}}$

$$y'=\frac{2(x+1)^{1/2}-(2x+1)\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}(1)}{\left[(x+1)^{1/2}\right]^2}=\frac{\frac{2(x+1)^{\overset{1}{\cancel{1/2}}}}{\cancel{(x+1)^{-1/2}}}-\frac{1}{2}(2x+1)\cancel{(x+1)^{-1/2}}}{\cancel{(x+1)^{-1/2}}}$$
$$x+1$$

GCF $=(x+1)^{-1/2}$

$$y'=\frac{(x+1)^{-1/2}\left[2(x+1)-\frac{1}{2}(2x+1)\right]}{x+1}=\frac{2x+2-x-\frac{1}{2}}{(x+1)^{1/2}(x+1)^1}=\frac{\frac{2\cdot x}{2\cdot 1}+\frac{3}{2}}{(x+1)^{3/2}}$$

LCD $=2$

$$y'=\frac{\frac{2x+3}{2}}{(x+1)^{3/2}}=\frac{2x+3}{2}\div(x+1)^{3/2}=\frac{2x+3}{2}\times\frac{1}{(x+1)^{3/2}}=\boxed{\frac{2x+3}{2(x+1)^{3/2}}}$$