Implicit Differentiation

Previously all expressions were set equal to *y* (Explicit Differentiation).

$f(x)=3x^2+7x-5$

$f'(x)=6x+7$

$\frac{dy}{dx}=6x+7$

$y'=6x+7$

$f(x)=\frac{3x-1}{5x-1}$

$f'(x)=\frac{2}{(5x-1)^2}$

$\frac{dy}{dx}=\frac{2}{(5x-1)^2}$

$y'=\frac{2}{(5x-1)^2}$

$f(x)=\sqrt{(x-1)^3}$

$f'(x)=\frac{3\sqrt{x-1}}{2}$

$\frac{dy}{dx}=\frac{3\sqrt{x-1}}{2}$

$y'=\frac{3\sqrt{x-1}}{2}$

Not all equations are set equal to *y*.

$x^2+3y^2+7x=4$ $\qquad$ $x^2+2xy+y^2+7x=3$

Treat *y* as some unknown but differentiable function of *x*.

Examples:

1. Find *y'*.

a) $x^3+y^3+5y=7$

$3x^2+3y^2y'+5y'=0$

$-3x^2 \qquad\qquad -3x^2$

$3y^2y'+5y'=-3x^2$

$y'(3y^2+5)=-3x^2$

$3y^2+5 \qquad\qquad 3y^2+5$

$$y'=\frac{-3x^2}{3y^2+5}$$

b) $x^3-3x^2y^4+4y^3=6x+1$

$3x^2-3(2xy^4+x^2\cdot 4y^3y')+12y^2y'=6+0$

$3x^2-6xy^4-12x^2y^3y'+12y^2y'=6$

$-3x^2+6xy^4 \qquad\qquad -3x^2+6xy^4$

$$-12x^2y^3y' + 12y^2y' = 6 - 3x^2 + 6xy^4$$

$$y'(-12x^2y^3 + 12y^2) = 6 - 3x^2 + 6xy^4$$

$$y' = \frac{6 - 3x^2 + 6xy^4}{-12x^2y^3 + 12y^2}$$

GCF = 3

$$y' = \frac{3(2 - x^2 + 2xy^4)}{3(-4x^2y^3 + 4y^2)} = \boxed{\frac{2 - x^2 + 2xy^4}{-4x^2y^3 + 4y^2}}$$

c) $2xy + y^2 = x + y$

$$2(1y + x(1)y') + 2yy' = 1 + 1y'$$

$$2y + 2xy' + 2yy' = 1 + y'$$

$$2xy' + 2yy' - y' = 1 - 2y$$

$$y'(2x + 2y - 1) = 1 - 2y$$

$$\boxed{y' = \frac{1 - 2y}{2x + 2y - 1}}$$

2. Find the slope of $x^2 + xy + 2y^2 = 28$ at $(2,3)$.

$$2x + 1y + x(1)y' + 4yy' = 0$$

$$2x + y + xy' + 4yy' = 0$$

$$xy' + 4yy' = -2x - y$$

$$y'(x + 4y) = -2x - y$$

$$y' = \frac{-2x - y}{x + 4y}\Bigg|_{(2,3)} = \frac{-2(2) - 3}{2 + 4(3)} = \frac{-4 - 3}{2 + 12} = \frac{-7}{14} = \boxed{-\frac{1}{2}}$$

3. Find the slope of $2y^2 + 2xy = 1$ at $x = \frac{1}{2}$.

$$4yy' + 2(1y + x(1)y') = 0$$

$x = \frac{1}{2}$ $\quad 2y^2 + 2xy = 1$

3. Find the slope of $2y^2 + 2xy = 1$ at $x = \frac{1}{2}$.

$4yy' + 2(1y + x(1)y') = 0$

$4yy' + 2y + 2xy' = 0$

$4yy' + 2xy' = -2y$

$y'(4y + 2x) = -2y$

$y' = \frac{-2y}{4y+2x} = \frac{-2y}{2(2y+x)} = \frac{-y}{2y+x}$

$x = \frac{1}{2}$ $\quad 2y^2 + 2xy = 1$

$2y^2 + 2(\frac{1}{2})y = 1$

$2y^2 + y = 1$

$2y^2 + y - 1 = 0$

$(2y-1)(y+1) = 0$

$2y - 1 = 0 \quad y + 1 = 0$

$y = 1/2 \quad y = -1$

$(1/2, 1/2) \quad (1/2, -1)$

$y' = \frac{-y}{2y+x}\Big|_{(1/2,1/2)} = \frac{-\frac{1}{2}}{2(\frac{1}{2})+\frac{1}{2}} = \frac{-\frac{1}{2}}{1+\frac{1}{2}} = \frac{-\frac{1}{2}}{\frac{3}{2}} = \boxed{-\frac{1}{3}}$

$y' = \frac{-y}{2y+x}\Big|_{(1/2,-1)} = \frac{-(-1)}{2(-1)+\frac{1}{2}} = \frac{1}{-2+\frac{1}{2}} = \frac{1}{-\frac{3}{2}} = \boxed{-\frac{2}{3}}$

4. Find the slope of $y^3 + 2x = 7y$ at $y = 1$.

$3y^2y' + 2 = 7y'$

$-2 \quad -7y'$

$3y^2y' - 7y' = -2$

$y'(3y^2 - 7) = -2$

$3y^2 - 7 \quad 3y^2 - 7$

$y' = \frac{-2}{3y^2-7}\Big|_{y=1} = \frac{-2}{3(1)^2-7} = \frac{-2}{3-7} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$