Related Rates

1. A spherical balloon is being inflated at a constant rate of 100 cubic feet/minute. How fast is the radius of the balloon increasing at the instant when the radius in 5 feet?

$\frac{dV}{dt} = 100 \text{ ft}^3/\text{min}$

$R = 5 \text{ feet}$

$\frac{dR}{dt}$

$V = \frac{4}{3}\pi R^3$

$\frac{dV}{dt} = \frac{4}{3}\cdot\pi\cdot 3R^2\frac{dR}{dt}$

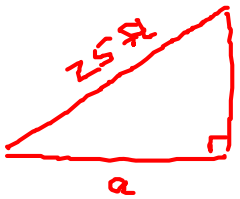
$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$

$100 = 4\pi(5)^2\frac{dR}{dt}$

$\frac{100}{100\pi} = \frac{100\pi}{100\pi}\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{1}{\pi} \text{ ft/min}}$

2. A ladder is 25 feet long with one end againt a vertical wall and the other end on the ground. The lower end is being moved away from the wall at a rate of 4 feet/second. How fast is the top of the ladder descending when the bottom of the ladder is 7 feet from the wall?



$\frac{db}{dt}$

$\frac{da}{dt} = 4 \text{ ft/sec}$

$a = 7$

$a^2 + b^2 = c^2$

$7^2 + b^2 = 25^2$

$49 + b^2 = 625$

$b^2 = 576 \quad b = 24$

$a^2 + b^2 = c^2$

$2a\cdot\frac{da}{dt} + 2b\cdot\frac{db}{dt} = 2c\cdot\frac{dc}{dt}$

$2(7)(4) + 2(24)\frac{db}{dt} = 2(25)(0)$

$56 + 48\frac{db}{dt} = 0$

$-56 \qquad -56$

$\frac{48}{48}\frac{db}{dt} = \frac{-56}{48}$

$\boxed{\frac{db}{dt} = \frac{-7}{6} \text{ ft/sec}}$

3. The radius of a circular plate is increasing at a rate of 2 centimeters/second. How fast is the area of the plate changing when the radius is 10 cm?

$\frac{dr}{dt} = 2 \text{ cm/sec}$

$\frac{dA}{dt}$

$R = 10 \text{ cm}$

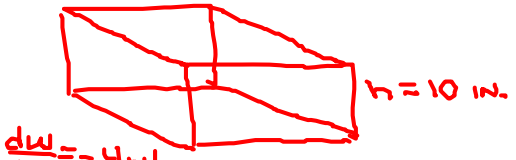
$A = \pi R^2$

$\frac{dA}{dt} = \pi \cdot 2R \cdot \frac{dR}{dt}$

$\frac{dA}{dt} = \pi \cdot 2(10)(2)$

$\boxed{\frac{dA}{dt} = 40\pi \text{ cm}^2/\text{sec}}$

4. The height of a rectangular box is 10 inches. If the length increases at a rate of 2 inches/second and its width decreases at a rate of 4 inches/second, at what rate is the volume of the box changing when the length is 8 inches and the width is 6 inches?



$\frac{dw}{dt} = -4 \text{ in/s}$

$\frac{dL}{dt} = 2 \text{ in/sec}$

$w = 6$

$L = 8$

$\frac{dV}{dt}$

$V = LWH$

$V = 10LW$

$\frac{dV}{dt} = 10\left(1 \cdot \frac{dL}{dt} w + L \cdot 1 \cdot \frac{dw}{dt}\right)$

$\frac{dV}{dt} = 10\left(2 \cdot 6 + 8 \cdot (-4)\right)$

$\frac{dV}{dt} = 10(12 - 32)$

$\boxed{\frac{dV}{dt} = -200 \text{ in}^3/\text{sec}}$

5. An ice cream cone 5 inches high and 2 inches in diameter is leaking ice cream from a hole in the bottom at a rate of $\frac{1}{3}$ cubic inches/minute. At what rate is the level of the ice cream falling when the height of the cone measures 3 inches?

$h = 5 \quad R = 1$

$h = 3 \quad R = \frac{3}{5}$

$\frac{5}{3} = \frac{1}{R}$

$5R = 3$

$R = \frac{3}{5}$ in

$V = \frac{1}{3}\pi R^2 h$

$V = \frac{1}{3}\pi \left(\frac{3}{5}\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{9}{25} h$

$V = \frac{3\pi}{25} h$

$\frac{dV}{dt} = -\frac{1}{3}$ in^3/min

$\frac{dh}{dt}$

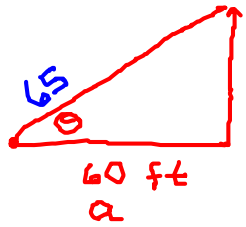
$h = 3$

$(1)\ \frac{dV}{dt} = \frac{3\pi}{25}(1)\frac{dh}{dt}$

$\frac{25}{3\pi} \cdot -\frac{1}{3} = \frac{3\pi}{25} \cdot \frac{dh}{dt} \cdot \frac{25}{3\pi}$

$\boxed{\frac{dh}{dt} = -\frac{25}{9\pi} \text{ in/min}}$

6. A balloon rises at a rate of 8 feet per second from a point 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground.



$\frac{dh}{dt} = 8$ ft/sec

$h = 25$ ft

$\frac{d\theta}{dt}$

$a^2 + b^2 = c^2$

$60^2 + 25^2 = c^2$

$3600 + 625 = c^2$

$4225 = c^2$

$c = 65$

$\cos\theta = \frac{60}{65}$

$\tan\theta = \frac{h}{a}$

$\tan\theta = \frac{h}{60}$

$\tan\theta = \frac{1}{60} \cdot h$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{60} \cdot (1)\frac{dh}{dt}$

$\frac{\sec^2\theta}{\sec^2\theta} \frac{d\theta}{dt} = \frac{1}{60} \cdot \frac{2}{8} \cdot \frac{1}{\sec^2\theta}$

$\frac{d\theta}{dt} = \frac{2}{15} \cdot \cos^2\theta$

$\frac{d\theta}{dt} = \frac{2}{15} \cdot \left(\frac{60}{65}\right)^2 = \frac{7200}{63{,}375}$

$\boxed{\frac{d\theta}{dt} = .113 \text{ deg/sec}}$

7. Let $y = \sin x$, $x = \frac{\pi}{6}$ and $\frac{dx}{dt} = 2$. Find $\frac{dy}{dt}$.

$\rightarrow y = \sin x$

$x = \frac{\pi}{6}$

$\frac{dx}{dt} = 2$

$y = \sin x$

$(1)\ \frac{dy}{dt} = \cos x \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = \left(\cos\frac{\pi}{6}\right)(2)$

$\frac{dy}{dt} = \frac{\sqrt{3}}{2} \cdot 2$

$\boxed{\frac{dy}{dt} = \sqrt{3}}$