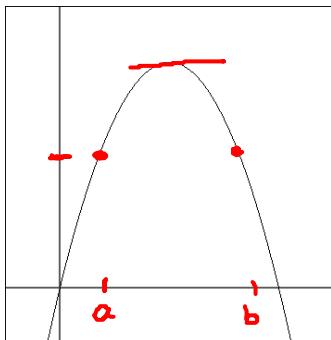
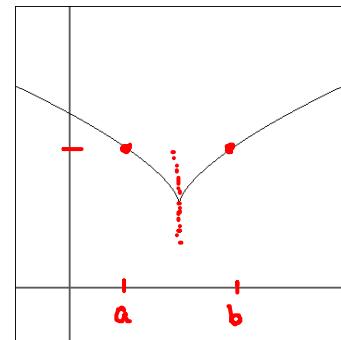
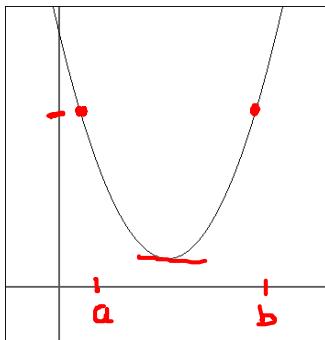


Rolle's Theorem

Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b) . If $f(a)=f(b)$ then there is at least one number c in (a,b) such that $f'(c)=0$.



*Rolle's Theorem
applies*

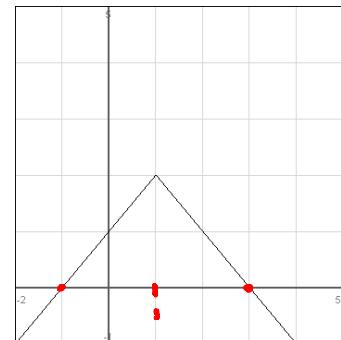


*Rolle's Theorem
does not apply*

- Explain why Rolle's Theorem does not apply on the closed interval $[a,b]$.

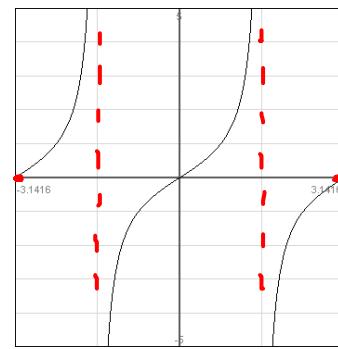
a) $f(x) = -|x-1| + 2 \quad [-1,3]$

*$f(x)$ is not differentiable
at $x=1$*



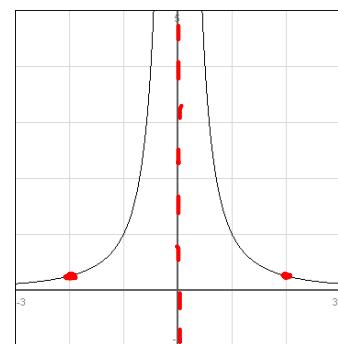
b) $f(x) = \tan x$ $[-\pi, \pi]$

$f(x)$ is not continuous
at $x = -\pi/2$ and $\pi/2$



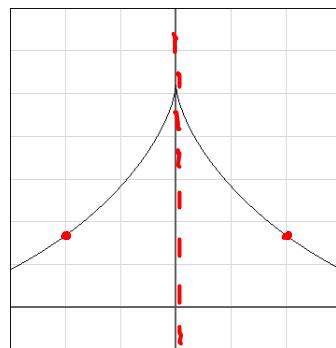
c) $f(x) = \frac{1}{x^2}$ $[-2, 2]$

$f(x)$ is not continuous
at $x = 0$



d) $f(x) = \sqrt{(3-x^{\frac{2}{3}})^3}$ $[-2, 2]$

$f(x)$ is not differentiable at $x=0$



calculator: $f(x) = (3-x^{\frac{2}{3}})^{\frac{3}{2}}$
 $\rightarrow (3-x^{\frac{2}{3}})^{\frac{3}{2}} \cdot (3/x)$
 $\rightarrow (3-(-x)^{\frac{2}{3}})^{\frac{3}{2}} \cdot (3/x)$

2. Find the x -intercepts of the function and show that $f'(x)=0$ at some point between the x -intercepts.

$$f(x) = \frac{x^3}{3} - 3x$$

$$\frac{x^3}{3} - 3x = 0 \quad L.C.D = 3$$

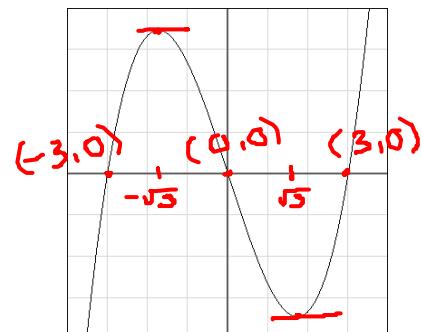
$$\frac{x^3}{3} - \frac{9x}{3} = 0$$

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x(x+3)(x-3) = 0$$

$$\boxed{x=0} \quad \boxed{x=-3} \quad \boxed{x=3}$$



$$f(x) = \frac{x^3}{3} - 3x$$

$$f'(x) = \frac{3x^2}{3} - 3$$

$$f'(x) = x^2 - 3$$

$$x^2 - 3 = 0$$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \pm 1.7$$

3. Determine whether Rolle's Theorem can be applied to f on the closed interval $[a,b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a,b) such that $f'(c)=0$.

a) $f(x) = x^3 - x^2 - 5x - 3 \quad [-1,3]$

continuous $[-1,3] \checkmark$

$f'(x) = 3x^2 - 2x - 5$

differentiable $(-1,3) \checkmark$

$$\frac{3x^2 - 2x - 5}{3} = 0$$

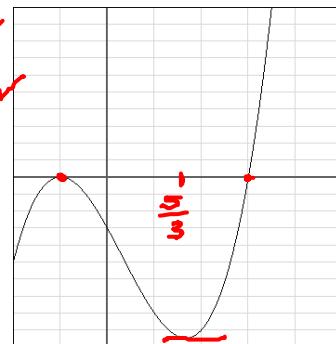
$$x^2 - 2x - 15 = 0$$

$$(x - \frac{5}{3})(x + 3) = 0$$

$$(x - \frac{5}{3})(x + 1) = 0$$

$$x - \frac{5}{3} = 0 \quad x + 1 = 0$$

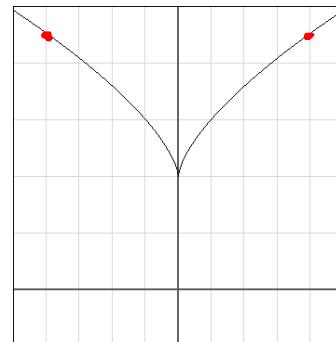
$$\boxed{x = \frac{5}{3}} \quad x = -1$$



b) $f(x) = x^{\frac{2}{3}} + 2 \quad [-4,4]$ continuous $[-4,4] \checkmark$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

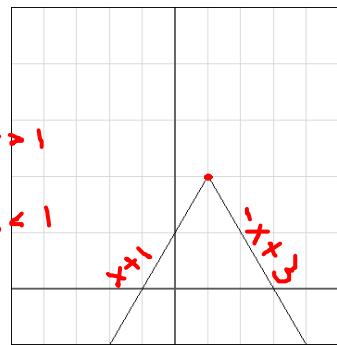
not differentiable at $x=0$



Rolle's Theorem does not apply because
 $f(x)$ is not diff. at $x=0$

c) $f(x) = 2 - |x-1| \quad [-4,4] \quad \text{continuous } [-4,4] \checkmark$

$$f(x) = \begin{cases} 2 - (x-1) = 2-x+1 = -x+3, & x \geq 1 \\ 2 + (x-1) = 2+x-1 = x+1, & x < 1 \end{cases}$$



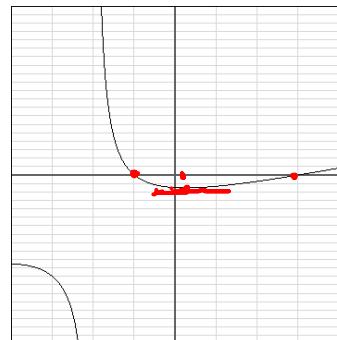
$$f'(x) = \begin{cases} -1, & x > 1 \\ 1, & x < 1 \end{cases}$$

Rolle's Theorem does not apply because
 $f(x)$ is not diff. at $x=1$

d) $f(x) = \frac{x^2 - 2x - 3}{x+2} \quad [-1,3] \quad \text{continuous } [-1,3] \checkmark$

$$f'(x) = \frac{(2x-2)(x+2) - (1)(x^2 - 2x - 3)}{(x+2)^2}$$

$$f'(x) = \frac{2x^2 + 4x - 2x - 4 - x^2 + 2x + 3}{(x+2)^2}$$



$$f'(x) = \frac{x^2 + 4x - 1}{(x+2)^2} \quad \text{differentiable } (-1,3) \checkmark$$

$$x^2 + 4x - 1 = 0 \quad a=1 \quad b=4 \quad c=-1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)} = \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2}$$

$$x = -2 \pm \sqrt{5}$$

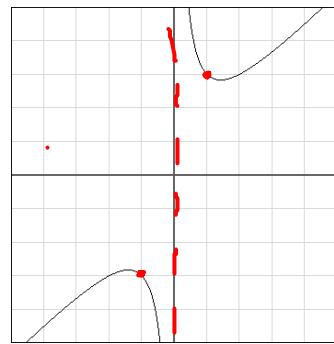
$$x = .236, -4.236$$

$$\boxed{x = .236}$$

e) $f(x) = \frac{x^2 + 2}{x}$ $[-1, 1]$

continuous $[-1, 1]$ X

Vertical Asymptote at $x=0$



Rolle's Theorem does not apply because $f(x)$ is not cont. at $x=0$

f) $f(x) = \cos x$ $[0, 2\pi]$

continuous $[0, 2\pi]$ ✓

$f'(x) = -\sin x$ differentiable $(0, 2\pi)$ ✓

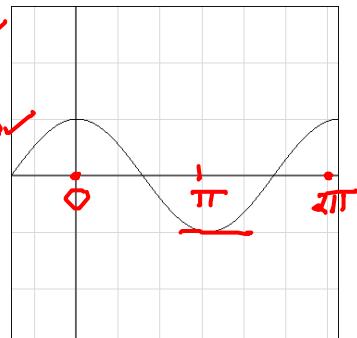
$-\sin x = 0$

$\sin x = 0$

$x = 0^\circ, 180^\circ, 360^\circ$

$x = 0, \pi, 2\pi$

$x = \pi$



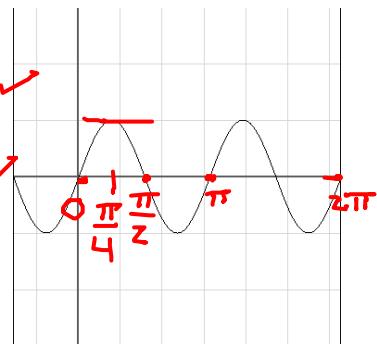
$$g) f(x) = \sin 2x \quad \left[0, \frac{\pi}{2}\right]$$

continuous $[0, \frac{\pi}{2}]$ ✓

$$f'(x) = 2 \cos 2x$$

differentiable $(0, \frac{\pi}{2})$ ✓

$$\frac{2 \cos 2x}{2} = 0$$



$$\cos 2x = 0$$

$$x = \frac{\pi}{4}$$

