Finding Volumes of Solids Using the Disk Method

Vertical Disks: Volume = $\int_a^b \pi\left(\text{Radius Function}\right)^2 dx$

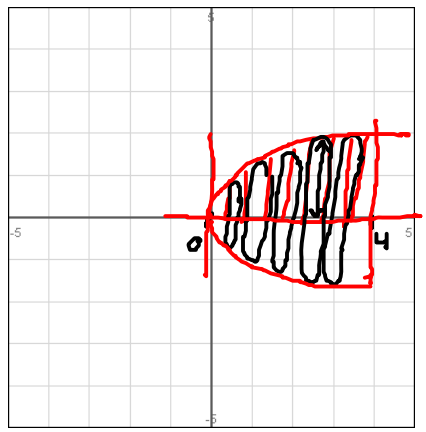
Horizontal Disks: Volume = $\int_c^d \pi\left(\text{Radius Function}\right)^2 dy$

1. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$, $0 \le x \le 4$ and the $x-axis$ about the $x-axis$.

$$\int_a^b \pi R^2 dx \qquad R = \sqrt{x} - 0 = \sqrt{x}$$

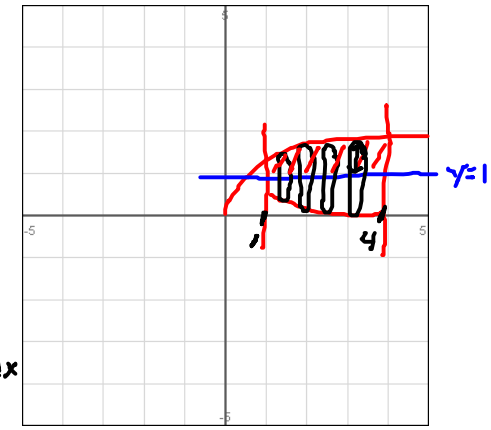
$$\int_0^4 \pi(\sqrt{x})^2 dx = \int_0^4 \pi x\, dx = \pi \frac{x^2}{2}\Big|_0^4$$

$$\pi\left[\frac{4^2}{2} - \frac{0^2}{2}\right] = \pi(8) = \boxed{8\pi}$$



2. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $x=1$ and $x=4$ about the line $y=1$.

$$\int_a^b \pi R^2\,dx \qquad R=\sqrt{x}-1$$



$$\int_1^4 \pi(\sqrt{x}-1)^2\,dx = \int_1^4 \pi(\sqrt{x}-1)(\sqrt{x}-1)\,dx$$

$$\int_1^4 \pi\left(x-\sqrt{x}-\sqrt{x}+1\right)dx = \int_1^4 \pi\left(x-2\sqrt{x}+1\right)dx$$

$$\int_1^4 \pi\left(x-2x^{\frac{1}{2}}+1\right)dx$$

$$\pi\left[\frac{x^2}{2}-\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}+x\right]\Bigg|_1^4 = \pi\left[\frac{x^2}{2}-\frac{4}{3}x^{3/2}+x\right]_1^4 = \pi\left[\left(\frac{4^2}{2}-\frac{4}{3}\cdot 8+4\right)-\left(\frac{1}{2}-\frac{4}{3}\cdot 1+1\right)\right]$$

$$\pi\left[8-\frac{32}{3}+4-\frac{1}{2}+\frac{4}{3}-1\right] = \pi\left[\frac{11\cdot 6}{1\cdot 6}-\frac{28\cdot 2}{3\cdot 2}-\frac{1\cdot 3}{2\cdot 3}\right] = \pi\left[\frac{66-56-3}{6}\right] = \boxed{\frac{7\pi}{6}}$$

LCD = 6

3. The region between the curve $x = \frac{2}{y}$, $1 \le y \le 4$ and the $y-axis$ is revolved about the $y-axis$ to generate a solid.

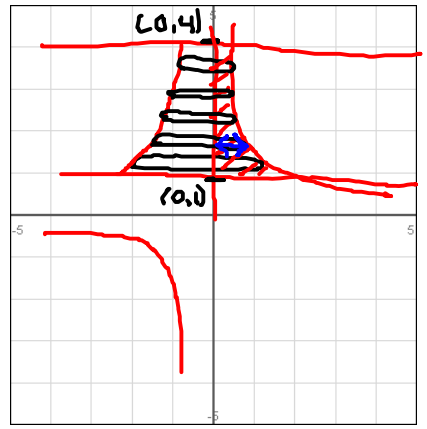
Find the volume.

$$\frac{x}{1} = \frac{2}{y}$$

$$\frac{xy}{x} = \frac{2}{x}$$

$$y = \frac{2}{x}$$

$$\int_c^d \pi R^2 \, dy \qquad R = \frac{2}{y} - 0$$



$$\int_1^4 \pi \left(\frac{2}{y}\right)^2 dy = \int_1^4 \pi \cdot \frac{4}{y^2} \, dy$$

$$\int_1^4 4\pi y^{-2} \, dy = 4\pi \cdot \frac{y^{-1}}{-1} \Big|_1^4 = \frac{-4\pi}{y} \Big|_1^4$$

$$-4\pi \left[\frac{1}{4} - \frac{1 \cdot 4}{1 \cdot 4}\right] = -4\pi \left[\frac{1-4}{4}\right] = -4\pi \cdot \frac{-3}{4} = \boxed{3\pi}$$

LCD = 4