Inverse Trigonometric Functions - Differentiation

Differentiation

$$\frac{d}{dx}\arcsin u = \frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx} \qquad \frac{d}{dx}\operatorname{arc}\csc u = -\frac{1}{|u|\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\arccos u = -\frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx} \qquad \frac{d}{dx}\operatorname{arc}\sec u = \frac{1}{|u|\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\arctan u = \frac{1}{1+u^2}\cdot\frac{du}{dx} \qquad \frac{d}{dx}\operatorname{arc}\cot u = -\frac{1}{1+u^2}\cdot\frac{du}{dx}$$

Directions: For questions 1 through 6, verify each differentiation formula.

1. $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

$y = \arcsin x$

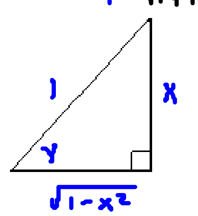
$\sin y = x$ *

$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$

$\sin y = \frac{x}{1}$ opp / hyp



$a^2+b^2=c^2$

$x^2+b^2=1^2$

$x^2+b^2=1$

$b^2=1-x^2$

$b=\sqrt{1-x^2}$

$\cos y = \frac{\sqrt{1-x^2}}{1}$

$\cos y = \sqrt{1-x^2}$

2. $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$

$y = \arccos x$

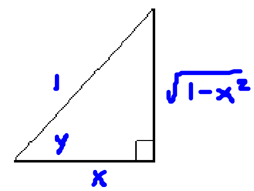
$\cos y = x$ *

$-\sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{-\sin y}$

$\boxed{\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}}$

$\cos y = \frac{x}{1}$ adj / hyp



$a^2+b^2=c^2$

$x^2+b^2=1^2$

$x^2+b^2=1$

$b^2=1-x^2$

$b=\sqrt{1-x^2}$

$\sin y = \frac{\sqrt{1-x^2}}{1}$

$\sin y = \sqrt{1-x^2}$

3. $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

$y = \arctan x$

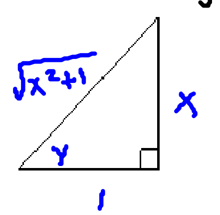
$\tan y = x$ *

$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\boxed{\frac{dy}{dx} = \frac{1}{x^2+1}}$

$\tan y = \frac{x}{1}$ opp / adj



$a^2+b^2=c^2$

$x^2+1^2=c^2$

$x^2+1=c^2$

$c=\sqrt{x^2+1}$

$\sec y = \frac{\sqrt{x^2+1}}{1}$

$\sec y = \sqrt{x^2+1}$

$\sec^2 y = (\sqrt{x^2+1})^2$

$\sec^2 y = x^2+1$

4. $\frac{d}{dx} \operatorname{arc}\csc x = -\frac{1}{|x|\sqrt{x^2-1}}$

$y = \operatorname{arccsc} x$

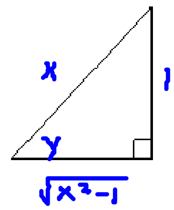
$\csc y = x$ *

$-\csc y \cot y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{-\csc y \cot y}$

$\boxed{\frac{dy}{dx} = \frac{1}{-x\sqrt{x^2-1}}}$

$\csc y = \frac{x}{1}\frac{\text{hyp}}{\text{opp}}$



$a^2+b^2=c^2$

$1^2+b^2=x^2$

$1+b^2=x^2$

$b^2=x^2-1$

$b=\sqrt{x^2-1}$

$\csc y = \frac{x}{1}$ $\quad \cot y = \frac{\sqrt{x^2-1}}{1}$

$\csc y = x$ $\quad \cot y = \sqrt{x^2-1}$

5. $\frac{d}{dx} \operatorname{arc}\sec x = \frac{1}{|x|\sqrt{x^2-1}}$

$y = \operatorname{arcsec} x$

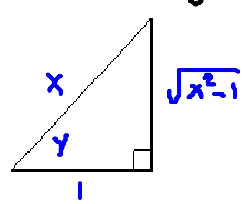
$\sec y = x$ *

$\sec y \tan y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$

$\boxed{\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}}$

$\sec y = \frac{x}{1}\frac{\text{hyp}}{\text{adj}}$



$a^2+b^2=c^2$

$1^2+b^2=x^2$

$1+b^2=x^2$

$b^2=x^2-1$

$b=\sqrt{x^2-1}$

$\sec y = \frac{x}{1}$ $\quad \tan y = \frac{\sqrt{x^2-1}}{1}$

$\sec y = x$ $\quad \tan y = \sqrt{x^2-1}$

6. $\frac{d}{dx}\operatorname{arc}\cot x = -\frac{1}{1+x^2}$

$y = \operatorname{arccot} x$

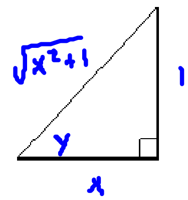
$\cot y = x$ *

$-\csc^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{-\csc^2 y}$

$\boxed{\frac{dy}{dx} = \frac{-1}{x^2+1}}$

$\cot y = \frac{x}{1} \frac{\text{adj}}{\text{opp}}$



$a^2+b^2=c^2$

$x^2+1^2=c^2$

$x^2+1=c^2$

$c=\sqrt{x^2+1}$

$\csc y = \frac{\sqrt{x^2+1}}{1}$

$\csc y = \sqrt{x^2+1}$

$\csc^2 y = \left(\sqrt{x^2+1}\right)^2$

$\csc^2 y = x^2+1$

Directions: For questions 7 through 10, find each derivative.

7. $y = \arcsin\frac{x}{2}$

$\frac{d}{dx}\arcsin u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$u = \frac{x}{2} \qquad \frac{du}{dx} = \frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2} = \frac{1}{\sqrt{\frac{4-x^2}{4}}} \cdot \frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{4-x^2}}{2}} \cdot \frac{1}{2} = \frac{2}{\sqrt{4-x^2}} \cdot \frac{1}{2} = \boxed{\frac{1}{\sqrt{4-x^2}} \text{ OR } \frac{\sqrt{4-x^2}}{4-x^2}}$

8. $y = \dfrac{\arccos 3x}{x}$

$$\frac{d}{dx}\arccos u = \frac{-1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$$

$$u = 3x \qquad \frac{du}{dx} = 3$$

$$\frac{dy}{dx} = \frac{\frac{-1}{\sqrt{1-(3x)^2}}\cdot 3\cdot x - 1\cdot\arccos 3x}{x^2} = \frac{\frac{-3x}{\sqrt{1-9x^2}} - \frac{\arccos 3x}{1}\cdot\sqrt{1-9x^2}}{x^2}$$

$LCD = \sqrt{1-9x^2}$

$$\frac{dy}{dx} = \frac{-3x-\sqrt{1-9x^2}\cdot\arccos 3x}{\sqrt{1-9x^2}}\cdot\frac{1}{x^2} = \boxed{\frac{-3x-\sqrt{1-9x^2}\cdot\arccos 3x}{x^2\sqrt{1-9x^2}}}$$

9. $y = x\arctan x^2$

$$\frac{d}{dx}\arctan u = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

$$u = x^2 \qquad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = 1\cdot\arctan x^2 + x\cdot\frac{1}{1+(x^2)^2}\cdot 2x = \frac{\arctan x^2}{1}\cdot(1+x^4) + \frac{2x^2}{1+x^4}$$

$LCD = 1+x^4$

$$\boxed{\frac{dy}{dx} = \frac{(1+x^4)\arctan x^2 + 2x^2}{1+x^4}}$$

10. $y = \tan\left(\operatorname{arc}\csc\sqrt{x}\right)$

$$\frac{d}{dx}\operatorname{arccsc} u = \frac{-1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

$$u = \sqrt{x} \qquad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \sec^2\left(\operatorname{arccsc}\sqrt{x}\right) \cdot \frac{-1}{|\sqrt{x}|\sqrt{(\sqrt{x})^2-1}} \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{-\sec^2\left(\operatorname{arccsc}\sqrt{x}\right)}{2x\sqrt{x-1}}}$$