Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

Directions: Evaluate each integral.

1. $\int xe^{2x}dx$

$u = x \qquad dv = e^{2x}dx$

$du = dx \qquad v = \frac{e^{2x}}{2}$

$x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}dx = \frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}dx = \frac{xe^{2x}}{2} - \frac{1}{4}\int e^{u}du$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{xe^{2x}}{2} - \frac{1}{4}e^{u} = \boxed{\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C}$

2. $\int \ln x\,dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$x \cdot \ln x - \int x \cdot \frac{1}{x}dx = x\ln x - \int 1\,dx = \boxed{x\ln x - x + C}$

3. $\int \frac{\ln x}{x^2}dx = -\frac{1}{x} \cdot \ln x + \int \frac{1}{x} \cdot \frac{1}{x}dx = -\frac{\ln x}{x} + \int \frac{1}{x^2}dx$

$u = \ln x \qquad dv = \frac{1}{x^2}dx$

$du = \frac{1}{x}dx \qquad v = -\frac{1}{x}$

$= -\frac{\ln x}{x} - \frac{1}{x} = \boxed{\frac{-\ln x - 1}{x} + C}$

4. $\int 2x\ln(x+1)\,dx$

$u = \ln(x+1)$ $\quad dv = 2x\,dx$

$du = \frac{1}{x+1}\,dx$ $\quad v = \frac{2x^2}{2} = x^2$

$$x^2 \cdot \ln(x+1) - \int x^2 \cdot \frac{1}{x+1}\,dx = x^2\ln(x+1) - \int \frac{x^2}{x+1}\,dx$$

Long Division:

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & x^2 \\ & -x^2 + x \\ \hline & -x \\ & +x+1 \\ \hline & 1 \end{array}$$

$x - 1 + \frac{1}{x+1}$

$$x^2\ln(x+1) - \int x - 1 + \frac{1}{x+1}\,dx = x^2\ln(x+1) - \left[\frac{x^2}{2} - x + \ln(x+1)\right]$$

$$\boxed{x^2\ln(x+1) - \frac{x^2}{2} + x - \ln(x+1) + C}$$

5. $\int_0^{\frac{\pi}{6}} \sec^2 x \ln(\cos x)\,dx$ $= \tan x \cdot \ln(\cos x) + \int_0^{\pi/6} \tan x \cdot \tan x\,dx$

$u = \ln(\cos x)$ $\quad dv = \sec^2 x\,dx$

$du = \frac{1}{\cos x} \cdot -\sin x\,dx = -\tan x\,dx$ $\quad v = \tan x$

$$\tan x\ln(\cos x) + \int_0^{\pi/6} \underbrace{\tan^2 x}_{\sec^2 x - 1}\,dx = \tan x\ln(\cos x) + \int_0^{\pi/6} \sec^2 x - 1\,dx$$

Pythagorean Identity: $\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$$\tan x\ln(\cos x) + \tan x - x\Big]_0^{\pi/6} = \left[\frac{1}{\sqrt{3}}\ln\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{3}} - \frac{\pi}{6}\right] - \left[0\ln(1) + 0 - 0\right]$$

$$= \frac{\sqrt{3}}{3}\ln\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{3} - \frac{\pi}{6} = \boxed{\frac{2\sqrt{3}\ln\left(\frac{\sqrt{3}}{2}\right) + 2\sqrt{3} - \pi}{6}}$$

6. $\int_0^{\pi} x^2 \sin x \, dx$ $= -x^2\cos x + \int_0^{\pi} 2x\cos x \, dx = -x^2\cos x + 2\int_0^{\pi} x\cos x \, dx$

$u = x^2$ $\quad dv = \sin x \, dx$

$du = 2x\,dx$ $\quad v = -\cos x$

$u = x$ $\quad dv = \cos x \, dx$

$du = dx$ $\quad v = \sin x$

$$-x^2\cos x + 2\left[x\sin x - \int_0^{\pi} \sin x \, dx\right]$$

$$-x^2\cos x + 2x\sin x - 2\int_0^{\pi} \sin x \, dx = -x^2\cos x + 2x\sin x + 2\cos x\Big]_0^{\pi}$$

$$\left[-\pi^2(-1) + 2\pi(0) + 2(-1)\right] - \left[0(1) + 0(0) + 2(1)\right]$$

$$\pi^2 - 2 - 2 = \boxed{\pi^2 - 4}$$

7. $\int_0^{\pi} e^x \sin x \, dx$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x dx \qquad v = -\cos x$

$$\int_0^{\pi} e^x \sin x \, dx = -e^x \cos x + \int_0^{\pi} e^x \cos x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x dx \qquad v = \sin x$

$$\int_0^{\pi} e^x \sin x \, dx = -e^x \cos x + \left[e^x \sin x - \int_0^{\pi} e^x \sin x \, dx \right]$$

$$\int_0^{\pi} e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int_0^{\pi} e^x \sin x \, dx$$

$$+\int_0^{\pi} e^x \sin x \, dx \qquad\qquad +\int_0^{\pi} e^x \sin x \, dx$$

$$\frac{2\int_0^{\pi} e^x \sin x \, dx}{2} = \frac{-e^x \cos x + e^x \sin x}{2}$$

$$\int_0^{\pi} e^x \sin x \, dx = \left. \frac{-e^x \cos x + e^x \sin x}{2} \right]_0^{\pi}$$

$$= \left[\frac{-e^{\pi}(-1) + e^{\pi}(0)}{2} \right] - \left[\frac{-e^0 (1) + e^0 (0)}{2} \right]$$

$$= \frac{e^{\pi}}{2} + \frac{1}{2} = \boxed{\frac{e^{\pi}+1}{2}}$$