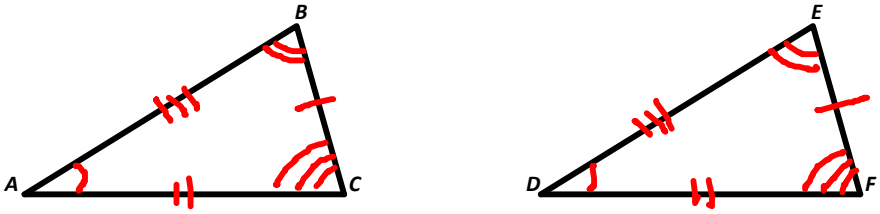
Congruent Triangles

Congruent Triangles - Two triangles are congruent if and only if their corresponding parts are congruent.

sides + angles

$\triangle ABC \cong \triangle DEF$



Third Angle Theorem - If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

40 + 80 = 120

180 − 120 = 60

Properties of Congruent Triangles

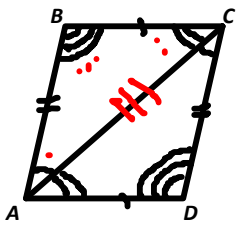
Reflexive Property - Every triangle is congruent to itself.

Symmetric Property - If $\triangle ABC \cong \triangle DEF$ then $\triangle DEF \cong \triangle ABC$.

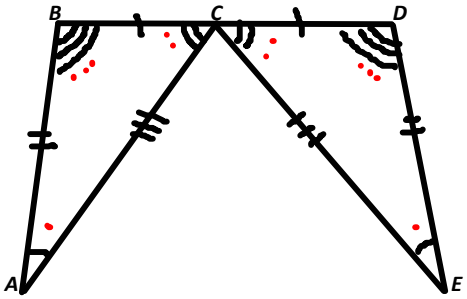
Transitive Property - If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHI$ then $\triangle ABC \cong \triangle GHI$.

1. Complete each congruence statement.

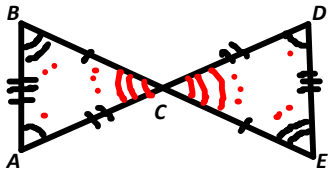
a) $\triangle ABC \cong$ $\triangle CDA$



b) $\triangle ABC \cong$ $\triangle EDC$

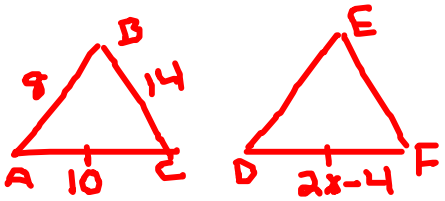


c) $\triangle ABC \cong$ $\triangle DEC$



2. $\triangle ABC \cong \triangle DEF$

a) If $AB = 8$, $BC = 14$, $AC = 10$ and $DF = 2x - 4$, find the value of x.



$AC = DF$

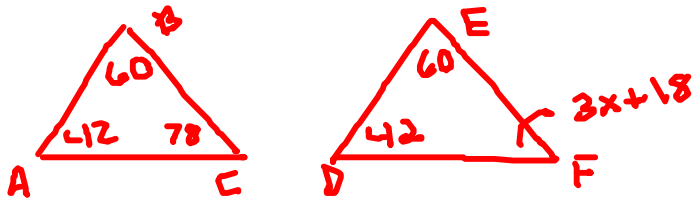
$10 = 2x - 4$

$+4 \quad +4$

$\frac{14}{2} = \frac{2x}{2}$

$x = 7$

b) If $m\measuredangle A = 42^\circ$, $m\measuredangle E = 60^\circ$ and $m\measuredangle F = 3x + 18$, find the value of x.



$60 + 42 = 102$ $\quad 180 - 102 = 78$

$m\angle C = m\angle F$

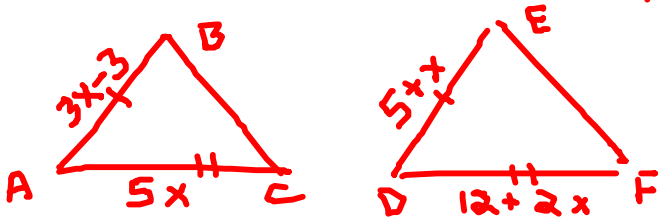
$78 = 3x + 18$

$-18 \quad -18$

$\frac{60}{3} = \frac{3x}{3}$

$x = 20$

3. If $\triangle ABC \cong \triangle DEF$, AB is three less than three times a number, DE is five more than a number, AC is five times a number and DF is twelve more than twice a number, find AB, AC, DE and DF.



$AB = DE$

$3x - 3 = 5 + x$

$-x \quad -x$

$2x - 3 = 5$

$+3 \quad +3$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$AC = DF$

$5x = 12 + 2x$

$-2x \quad -2x$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

$AB = 3(4) - 3 = 12 - 3 = 9$

$AB = 9$

$AC = 5(4) = 20$

$AC = 20$

$DE = 5 + 4 = 9$

$DE = 9$

$DF = 12 + 2(4) = 12 + 8 = 20$

$DF = 20$

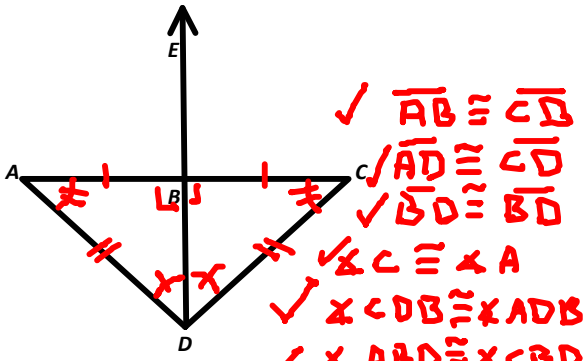
4. Given: $ED \perp AC$

B is the midpoint of $\overline{AC}$

$\overline{DA} \cong \overline{DC}$

$\overrightarrow{DE}$ bisects $\angle ADC$

Prove: $\triangle ABD \cong \triangle CBD$



✓ $\overline{AB} \cong \overline{CB}$

✓ $\overline{AD} \cong \overline{CD}$

✓ $\overline{BD} \cong \overline{BD}$

✓ $\angle C \cong \angle A$

✓ $\angle CDB \cong \angle ADB$

✓ $\angle ABD \cong \angle CBD$

Statement	Reason
1. $ED \perp AC$ •	1. Given
B is the midpoint of $\overline{AC}$ •	
$\overline{DA} \cong \overline{DC}$ •	
$\overrightarrow{DE}$ bisects $\angle ADC$ •	
2) $m\angle ABD = 90°$ $m\angle CBD = 90°$	2) Def. of Perpendicular
3) $m\angle ABD = m\angle CBD$	3) Substitution
4) $\angle ABD \cong \angle CBD$	4) Def. of $\cong$ Angles
5) $\overline{BA} \cong \overline{BC}$	5) Def. of midpoint
6) $\angle ADB \cong \angle CDB$	6) Def. of bisect
7) $\overline{BD} \cong \overline{BD}$	7) Reflexive
8) $\angle A \cong \angle C$	8) Third Angle Theorem
9) $\triangle ABD \cong \triangle CBD$	9) Def. of $\cong$ Triangles