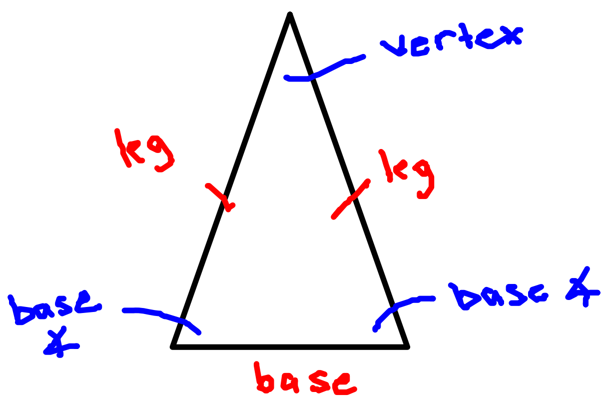
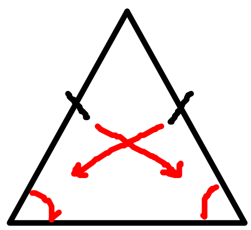
Isosceles Triangles

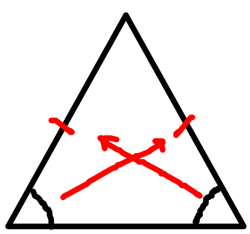
Isosceles Triangle - A triangle with at least two congruent sides.



Isosceles Triangle Theorem - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

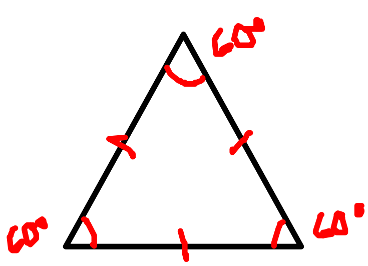


Isosceles Triangle Theorem Converse - If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



Corollary 1 - A triangle is equilateral if and only if it is equiangular.

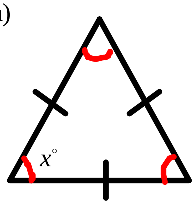
Corollary 2 - Each angle of an equilateral triangle measures 60˚.



$180° \div 3 = 60°$

1. Find the value of x.

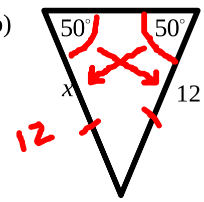
a)



$180 \div 3 = 60°$

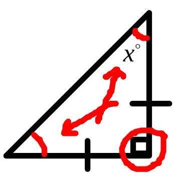
$\boxed{x = 60°}$

b)



$\boxed{x = 12}$

c)

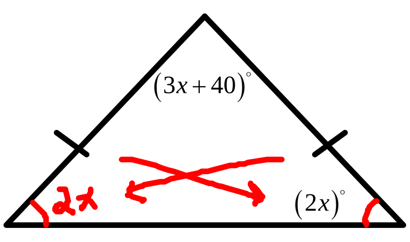


$180 - 90 = 90$

$90 \div 2 = 45$

$\boxed{x = 45°}$

d)



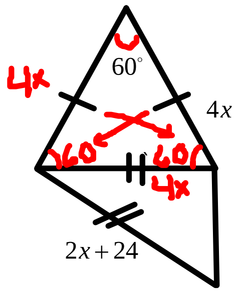
$3x + 40 + 2x + 2x = 180$

$7x + 40 = 180$

$-40 \quad -40$

$\frac{7x}{7} = \frac{140}{7}$

$\boxed{x = 20°}$

e) 

$180 - 60 = 120$

$120 \div 2 = 60$

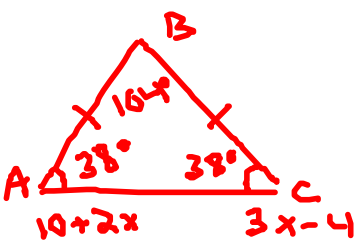
$4x = 2x + 24$

$-2x \quad -\cancel{2x}$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{24}{2}$

$\boxed{x = 12}$

2. In $\triangle ABC$, $\overline{AB} \cong \overline{BC}$, $m\measuredangle A$ is 10 more than twice a number and $m\measuredangle C$ is four less than three times the same number. Find $m\measuredangle B$.



$m\measuredangle A = 10 + 2x = 10 + 2(14) = 38°$

$m\measuredangle C = 3x - 4 = 3(14) - 4 = 38°$

$m\measuredangle A = m\measuredangle C$

$10 + \cancel{2x} = 3x - 4$

$-\cancel{2x} \quad -2x$

$10 = x \cancel{-4}$

$+4 \quad \cancel{+4}$

$14 = x$

$38 + 38 = 76$

$180 - 76 = 104$

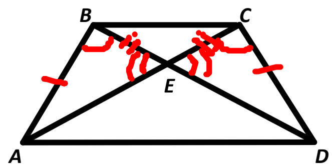
$\boxed{m\measuredangle B = 104°}$

3. Write a two-column proof for each.

a) Given: $\measuredangle ABD \cong \measuredangle DCA$

$\overline{BA} \cong \overline{CD}$

Prove: $\measuredangle BCA \cong \measuredangle CBD$

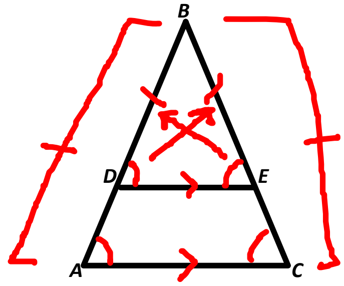


Statement	Reason
1. $\measuredangle ABD \cong \measuredangle DCA$, $\overline{BA} \cong \overline{CD}$	1. Given
2) $\measuredangle BEA \cong \measuredangle CED$	2) Vertical $\measuredangle$ Theorem
3) $\triangle BEA \cong \triangle CED$	3) AAS
4) $\overline{BE} \cong \overline{CE}$	4) CPCTC
5) $\triangle BEC$ is isos.	5) Def. of Isos. $\triangle$
6) $\measuredangle BCA \cong \measuredangle CBD$	6) Isos. $\triangle$ Theorem

b) Given: $\triangle ABC$ is an isosceles triangle and $\measuredangle B$ is the vertex

$\overline{DE} \parallel \overline{AC}$

Prove: $\triangle DBE$ is an isosceles triangle



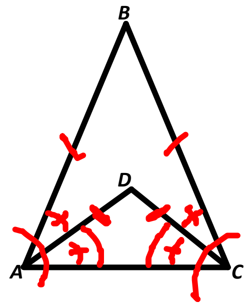
Statement	Reason
1. $\triangle ABC$ is an isosceles triangle and $\measuredangle B$ is the vertex $\overline{DE} \parallel \overline{AC}$	1. Given
2) $\overline{BA} \cong \overline{BC}$	2) Def of Isos. $\triangle$
3) $\measuredangle A \cong \measuredangle C$ *	3) Isos. $\triangle$ Theorem
4) $\measuredangle A \cong \measuredangle BDE$ * $\measuredangle C \cong \measuredangle BED$ ←	4) Corresponding $\measuredangle$'s Postulate
5) $\measuredangle C \cong \measuredangle BDE$ ←	5) Transitive
6) $\measuredangle BED \cong \measuredangle BDE$	6) Transitive
7) $\overline{BD} \cong \overline{BE}$ **	7) Isos. $\triangle$ Theorem Converse
8) $\triangle DBE$ is isos.	8) Def. of Isos. $\triangle$

c) Given: $\triangle ABC$ is an isosceles triangle and $\overline{AC}$ is the base

$\overline{DC}$ bisects $\measuredangle BCA$

$\overline{DA}$ bisects $\measuredangle BAC$

Prove: $\triangle ADC$ is an isosceles triangle



Statement	Reason
1. $\triangle ABC$ is an isosceles triangle and $\overline{AC}$ is the base $\overline{DC}$ bisects $\measuredangle BCA$ $\overline{DA}$ bisects $\measuredangle BAC$	1. Given
2) $\overline{BA} \cong \overline{BC}$	2) Def of isos Δ
3) $\measuredangle BAC \cong \measuredangle BCA$	3) Isos. Δ Theorem
4) $\measuredangle BAD \cong \measuredangle DAC$ $\measuredangle BCD \cong \measuredangle DCA$	4) Def of bisect
5) $m\measuredangle BAC = m\measuredangle BCA$ * $m\measuredangle BAD = \underline{m\measuredangle DAC}$ $m\measuredangle BCD = \underline{m\measuredangle DCA}$	5) Def of $\cong$ $\measuredangle$'s
6) $m\measuredangle BAC = m\measuredangle BAD + m\measuredangle DAC$ $m\measuredangle BCA = m\measuredangle BCD + m\measuredangle DCA$	6) $\measuredangle$ Addition Post.
7) $\underline{m\measuredangle BAD} + m\measuredangle DAC = \underline{m\measuredangle BCD} + m\measuredangle DCA$	7) substit.
8) $m\measuredangle DAC + m\measuredangle DAC = m\measuredangle DCA + m\measuredangle DCA$	8) substit.
9) $\frac{\cancel{2}}{\cancel{2}} \cdot m\measuredangle DAC = \frac{\cancel{2}}{\cancel{2}} \cdot m\measuredangle DCA$	9) Simplify
10) $m\measuredangle DAC = m\measuredangle DCA$	10) Divis. Prop.
11) $\measuredangle DAC \cong \measuredangle DCA$	11) Def. of $\cong$ $\measuredangle$'s
12) $\overline{DC} \cong \overline{DA}$	12) Isos. Δ Theorem Converse
13) ΔDAC is isos.	13) Def. of isos. Δ