Proving Triangles Using Coordinate Geometry

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

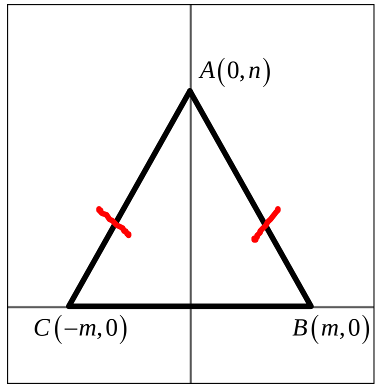
$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Directions: Write a coordinate proof.

1. Prove that $\triangle ABC$ is an isosceles triangle.



$A(0,N)$ $C(-m,0)$

x_1 y_1 x_2 y_2

$$d_{AC} = \sqrt{(-m-0)^2 + (0-N)^2}$$

$$= \sqrt{(-m)^2 + (-N)^2}$$

$$\boxed{= \sqrt{m^2 + N^2}}$$

$A(0,N)$ $B(m,0)$

x_1 y_1 x_2 y_2

$$d_{AB} = \sqrt{(m-0)^2 + (0-N)^2}$$

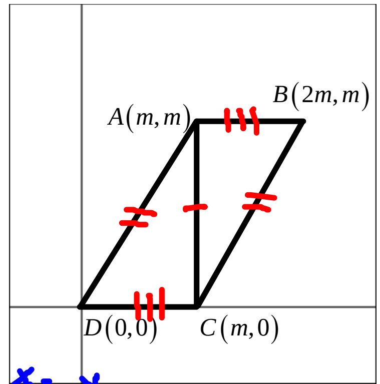
$$= \sqrt{(m)^2 + (-N)^2}$$

$$\boxed{= \sqrt{m^2 + N^2}}$$

Since $AC = AB$ then $\triangle ABC$ is isosceles.

2. Given: Coordinates of $\triangle ACD$ and $\triangle ACB$

Prove: $\triangle ACD \cong \triangle ACB$



* $\overline{AC} \cong \overline{CA}$ Reflexive Property

$A(m,m)$ $D(0,0)$ (x_1, y_1; x_2, y_2)

$d = \sqrt{(0-m)^2 + (0-m)^2}$

$= \sqrt{(-m)^2 + (-m)^2}$

$= \sqrt{m^2 + m^2}$

$= \sqrt{2m^2}$

$= |m|\sqrt{2}$

$B(2m,m)$ $C(m,0)$ (x_1, y_1; x_2, y_2)

$d = \sqrt{(m-2m)^2 + (0-m)^2}$

$= \sqrt{(-m)^2 + (-m)^2}$

$= \sqrt{m^2 + m^2} = \sqrt{2m^2} = |m|\sqrt{2}$

* $\overline{AD} \cong \overline{CB}$

$A(m,m)$ $B(2m,m)$ (x_1, y_1; x_2, y_2)

$d = \sqrt{(2m-m)^2 + (m-m)^2}$

$= \sqrt{(m)^2 + (0)^2}$

$= \sqrt{m^2}$

$= |m|$

$C(m,0)$ $D(0,0)$ (x_1, y_1; x_2, y_2)

$d = \sqrt{(0-m)^2 + (0-0)^2}$

$= \sqrt{(-m)^2 + (0)^2}$

$= \sqrt{m^2}$

$= |m|$

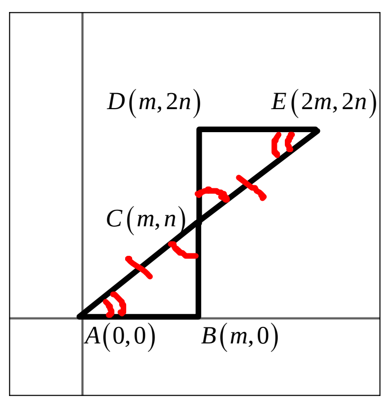
* $\overline{AB} \cong \overline{CD}$

Since $\overline{AC} \cong \overline{CA}$, $\overline{AD} \cong \overline{CB}$ and $\overline{AB} \cong \overline{CD}$ then $\triangle ACD \cong \triangle ACB$ by SSS Postulate.

3. Given: Coordinates of $\triangle ABC$ and $\triangle EDC$

Prove: $\triangle ABC \cong \triangle EDC$

SSS SAS
ASA AAS



* $\angle ACB \cong \angle ECD$ Vertical $\angle$ Theorem

$D(m,2n)$ $E(2m,2n)$ (x_1, y_1; x_2, y_2)

$m = \frac{2n-2n}{2m-m} = \frac{0}{m} = 0$

$\overline{DE} \parallel \overline{AB}$

$A(0,0)$ $B(m,0)$ (x_1, y_1; x_2, y_2)

$m = \frac{0-0}{m-0} = \frac{0}{m} = 0$

* $\angle A \cong \angle E$ Alternate Interior $\angle$ Theorem

$A(0,0)$ $C(m,n)$
$x_1\ y_1$ $x_2\ y_2$

$$d=\sqrt{(m-0)^2+(n-0)^2}$$
$$=\sqrt{(m)^2+(n)^2}$$
$$=\sqrt{m^2+n^2}$$

$E(2m,2n)$ $C(m,n)$
$x_1\ y_1$ $x_2\ y_2$

$$d=\sqrt{(m-2m)^2+(n-2n)^2}$$
$$=\sqrt{(-m)^2+(-n)^2}$$
$$=\sqrt{m^2+n^2}$$

* $\overline{AC}\cong\overline{EC}$

Since $\angle ACB\cong\angle ECD$, $\angle A\cong\angle E$ and $\overline{AC}\cong\overline{EC}$ then $\triangle ABC\cong\triangle EDC$ by ASA Postulate.