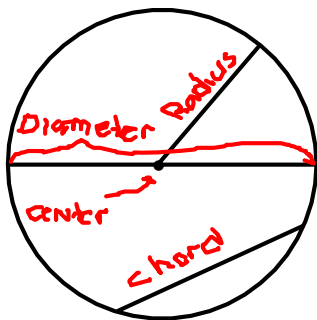# Circumference and Arc Length

Circumference - The distance around a circle.

$$C = 2\pi r$$

There are 360° in a circle.

There are 180° in a semicircle.

Central Angle - An angle whose vertex is at the center of the circle.

$$\text{Arc Length} = 2\pi r \cdot \frac{\text{central angle}}{360°}$$

1. Find the missing measures. Round your answers to the nearest tenth.

a) Radius = 6 feet

Diameter = $\boxed{12 \text{ feet}}$

Circumference =

$$C = 2\pi R$$
$$= 2(\pi)(6)$$
$$= \boxed{37.7 \text{ feet}}$$

b) Radius = $\boxed{22.0}$

Diameter = $\boxed{44.1}$

Circumference = 138.4 inches

$$C = 2\pi R$$
$$\frac{138.4}{\pi} = \frac{D\pi}{\pi}$$
$$D = 44.1$$

c) Radius = $\boxed{12.9}$

Diameter = 25.8 meters

Circumference =

$$C = 2\pi R$$
$$= 2\pi(12.9)$$
$$\approx \boxed{81.1}$$

d) Radius = $\dfrac{x}{3}$
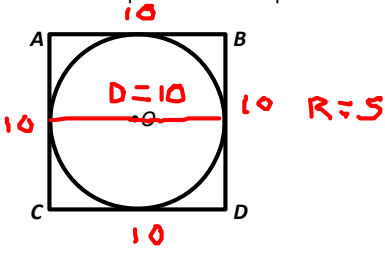
Diameter = $2\left(\dfrac{x}{3}\right) = \boxed{\dfrac{2x}{3}}$

Circumference =

$$C = 2\pi R$$
$$C = 2 \cdot \pi \cdot \frac{x}{3}$$
$$C = \boxed{\dfrac{2\pi x}{3}}$$

2. Find the exact circumference of each circle.

a) Circle *O* is inscribed in square *ABCD*. Each side of square *ABCD* is equal to 10.

10

A   B

D=10

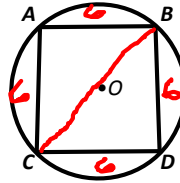10   ·O   10   R=5

C   D

10

$C = 2\pi R$
$C = 2\pi(5)$
$= \boxed{10\pi}$

b) Square *ABCD* is inscribed in Circle *O* and the length of each side of the square is equal to 6.

A   6   B

6   ·O   6

C   6   D

$6^2 + 6^2 = c^2$
$36 + 36 = c^2$
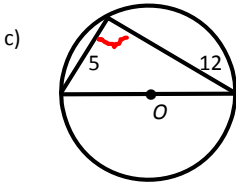$\sqrt{72} = \sqrt{c^2}$
$c = 6\sqrt{2}$
$D = 6\sqrt{2}$
$R = 3\sqrt{2}$

Circumference = $2\pi R$
$= 2\pi(3\sqrt{2})$
$= \boxed{6\sqrt{2}\pi}$

c)

5   12

O

$C = 2\pi R$
$C = 2\pi(6.5)$
$C = \boxed{13\pi}$

$5^2 + 12^2 = c^2$
$25 + 144 = c^2$
$\sqrt{169} = \sqrt{c^2}$
$C = 13$
$D = 13$
$R = 6.5$

3. If the wheel of a bicycle has a diameter of 2.5 feet, what is the total distance traveled after the wheel makes 10 revolutions?
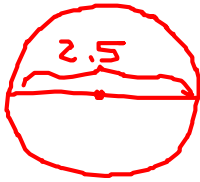
$D = 2.5$
$R = 1.25$
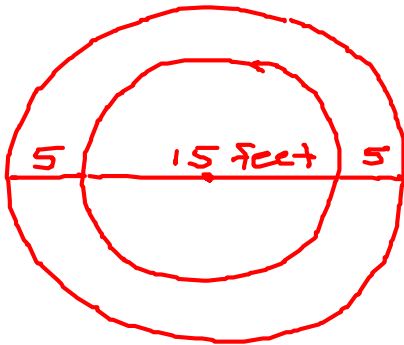
$C = 2\pi R$
$= 2\pi(1.25)$
$= 2.5\pi$ feet in 1 revolution

$2.5\pi \times 10 =$
25π feet in 10 revolutions

$\boxed{78.5 \text{ feet}}$

4. Frank wants to build a fence around a circular pool. If the diameter of the pool is 15 feet, and the fence is to be placed 5 feet from the edge of the pool, how many feet of fencing will be needed?
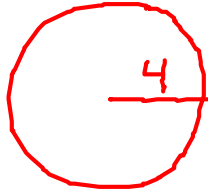
5   15 feet   5

$D = 25$ feet $R = 12.5$ feet

$C = 2\pi R$
$= 2\pi(12.5)$
$= 25\pi$
$= \boxed{78.5 \text{ feet}}$

5. If the radius of a circle is doubled, what effect does it have on the circumference?

$$C = 2\pi R$$
$$= 2\pi(2)$$
$$= 4\pi$$

$$C = 2\pi R$$
$$= 2\pi(4)$$
$$= 8\pi$$

The circumference is doubled

6. Find the length of each arc.

a)

3
40°

$R = 3$
$\angle = 40°$

b)

120°
7

$R = 7$
$\angle = 120°$
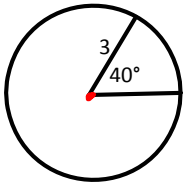
$$\text{Arc length} = 2\pi R \cdot \frac{\text{central} \angle}{360°}$$
$$= 2\pi(3) \cdot \frac{40°}{360°}$$
$$= \frac{6\pi}{9} \div \frac{3}{3} = \boxed{\frac{2\pi}{3}}$$
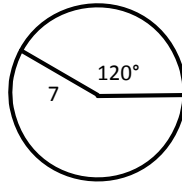
$$\text{Arc Length} = 2\pi R \cdot \frac{\text{central} \angle}{360°}$$
$$= 2\pi(7) \cdot \frac{120°}{360°}$$
$$= \boxed{\frac{14\pi}{3}}$$

7. Find the value of *x*.

a) *x* is the circumference

4.56

4.56

4.56

60° 60° 60° 60° 60° 60°

4.56

4.56

4.56
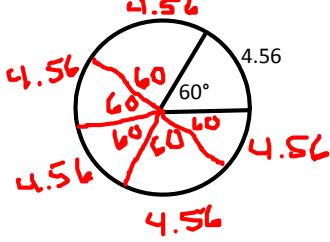
$C = 6(4.56)$

$= \boxed{27.36}$

b) *x* is the central angle

18

3.5 *x*

$R = 3.5$

Arc Length $= 18$

$x = x$
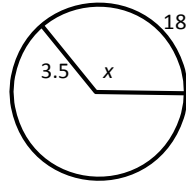
Arc Length $= 2\pi R \cdot \dfrac{x}{360°}$

$18 = 2\pi(3.5) \cdot \dfrac{x}{360}$

$\dfrac{18}{1} = \dfrac{7\pi x}{360}$

$\dfrac{7\pi x}{7\pi} = \dfrac{6480}{7\pi}$

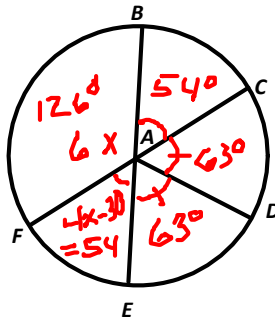$\boxed{x = 294.7}$

$6480 \div (7\pi) =$

8. Find the value of each if:

$\measuredangle CAD \cong \measuredangle EAD$

FAC and BAE are diameters

$\measuredangle BAF = 6x$

$\measuredangle EAF = 4x - 30$

Circle diagram with center A, points B (top), C, D (right side), E (bottom), F (left). Handwritten labels in red: $126°$, $54°$, $6x$, $63°$, $4x-30 = 54$, $63°$.

a) x

$6x + 4x - 30 = 180$
$10x - 30 = 180$
$\quad +30 \quad +30$

$\dfrac{10x}{10} = \dfrac{210}{10}$

$\boxed{x = 21}$

b) $\measuredangle BAF$

$6(21) = \boxed{126°}$

c) $\measuredangle FAE$

$4x - 30$
$4(21) - 30$
$84 - 30 = \boxed{54°}$

d) $\measuredangle EAD$

$180 - 54 = 126$
$126 \div 2 = \boxed{63°}$

e) $\measuredangle BAC$

$\boxed{54°}$

f) $\measuredangle CAD$

$\boxed{63°}$

9. Find the values of *x* and *y*.

300°

60

$(2x + 10)°$

9

$(y - 2)π$

$2x + 10 = 60$
$-10 \quad -10$

$\dfrac{2x}{2} = \dfrac{50}{2}$

$\boxed{x = 25}$

Arc length $= 2πR \cdot \dfrac{\text{central } \angle}{360°}$

$(y-2)π = 2π(9) \cdot \dfrac{60°}{360°}$

$(y-2)π = 18π \cdot \dfrac{60}{360}^{\,3}_{6}$

$(y-2)π = \dfrac{\overset{3}{18}π}{6}$

$(y-2)π = 3π$

$y - 2 = 3$
$\quad +2 \quad +2$

$\boxed{y = 5}$