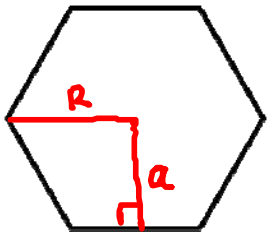
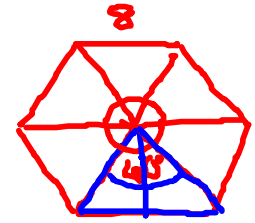
Areas of Regular Polygons



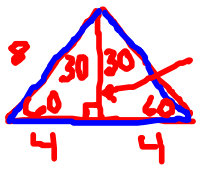
$$\text{Area} = \frac{1}{2} \bullet \text{Perimeter} \bullet \text{Apothem}$$

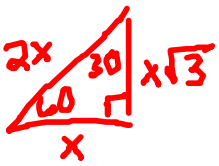
Directions: Find the area of each regular polygon.

1. A regular hexagon with a side length of 8 cm.



$360 \div 6 = 60°$





$A = \frac{1}{2} \cdot P \cdot a$

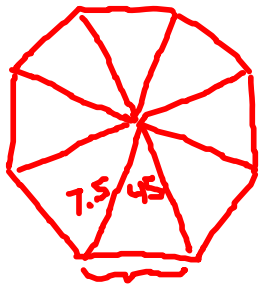
$P = 6 \cdot 8 = 48$

$a = 4\sqrt{3}$

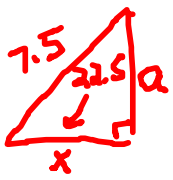
$A = \frac{1}{2} \cdot 24 \cdot 4\sqrt{3}$

$A = 96\sqrt{3}\ \text{cm}^2$

2. A regular octagon with a radius of 7.5 feet.



$360 \div 8 = 45°$



$\cos x = \frac{adj}{hyp}$

$\frac{\cos 22.5}{1} = \frac{a}{7.5}$

$a = 7.5 \cos 22.5$

$a = 6.93$

$\sin x = \frac{opp}{hyp}$

$\frac{\sin 22.5}{1} = \frac{x}{7.5}$

$x = 7.5 \sin 22.5$

$x = 2.87$

$side = 2(2.87) = 5.74$

$A = \frac{1}{2} \cdot P \cdot a$

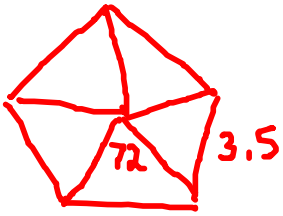
$a = 6.93$

$P = 8(5.74) = 45.92$

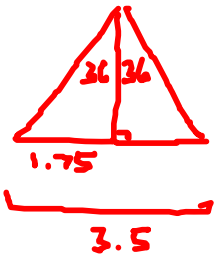
$A = \frac{1}{2}(45.92)(6.93)$

$= 159.10 \text{ feet}^2$

3. A regular pentagon with a side length of 3.5 inches.



$360 \div 5 = 72$



$\tan x = \frac{opp}{adj}$

$\frac{\tan 36}{1} = \frac{1.75}{a}$

$a \cdot \frac{\tan 36}{\tan 36} = \frac{1.75}{\tan 36}$

$a = \frac{1.75}{\tan 36}$

$a = 2.41$

$A = \frac{1}{2} P \cdot a$

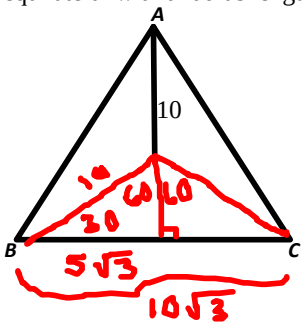
$P = 5(3.5) = 17.5$

$a = 2.41$

$A = \frac{1}{2}(17.5)(2.41)$

$A = 21.08 \text{ in}^2$

4. $\triangle ABC$ is equilateral with a radius length of 10.



360 ÷ 3 = 120

10, 60, 30, 5, $5\sqrt{3}$

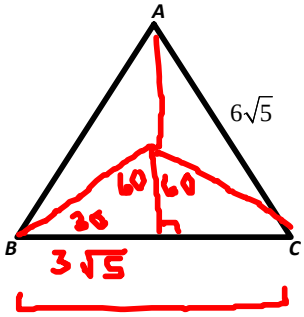
2x, 60, 30, x, $x\sqrt{3}$

$a = 5$ $\quad P = 3(10\sqrt{3}) = 30\sqrt{3}$

$A = \frac{1}{2} P \cdot a$

$A = \frac{1}{2} \cdot 30\sqrt{3} \cdot 5 = 75\sqrt{3}$ units^2

5. $\triangle ABC$ is equilateral with a side length of $6\sqrt{5}$.



360 ÷ 3 = 120

$2\sqrt{15}$, 60, $\sqrt{15}$, 30, $3\sqrt{5}$

2x, 60, 30, x, $x\sqrt{3}$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{5}}{\sqrt{3}}$

$x = \frac{3\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{15}}{\sqrt{9}} = \frac{3\sqrt{15}}{3}$

$x = \sqrt{15}$

$A = \frac{1}{2} \cdot P \cdot a$

$P = 3(6\sqrt{5}) = 18\sqrt{5}$

$a = \sqrt{15}$

$A = \frac{1}{2} \cdot 18\sqrt{5} \cdot \sqrt{15} = 9\sqrt{75} = 9\sqrt{25} \cdot \sqrt{3} = 9 \cdot 5\sqrt{3}$

$= 45\sqrt{3}$ units^2