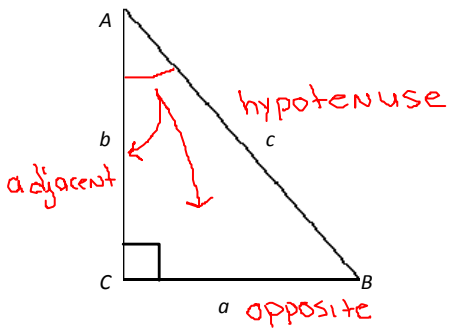
The Unit Circle



Definitions of Trigonometric Functions

"sine" $\sin\measuredangle = \dfrac{opposite}{hypotenuse}$ $\sin A = \dfrac{a}{c}$

"cosecant" $\csc\measuredangle = \dfrac{hypotenuse}{opposite}$ $\csc A = \dfrac{c}{a}$

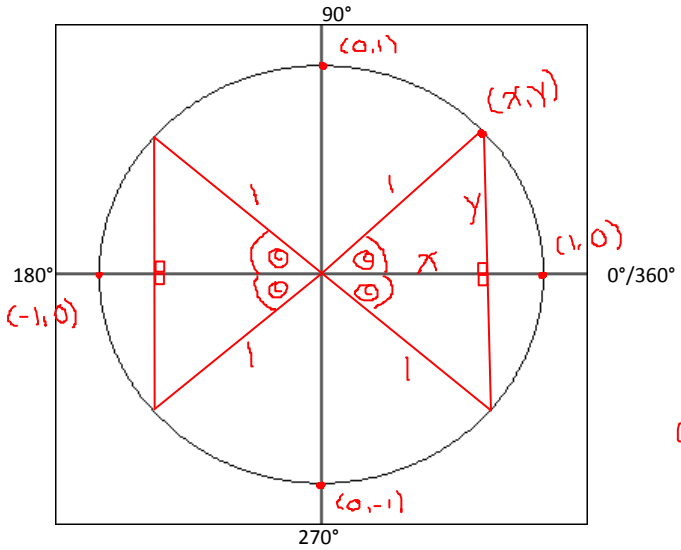
"cosine" $\cos\measuredangle = \dfrac{adjacent}{hypotenuse}$ $\cos A = \dfrac{b}{c}$

"secant" $\sec\measuredangle = \dfrac{hypotenuse}{adjacent}$ $\sec A = \dfrac{c}{b}$

"tangent" $\tan\measuredangle = \dfrac{opposite}{adjacent}$ $\tan A = \dfrac{a}{b}$

"cotangent" $\cot\measuredangle = \dfrac{adjacent}{opposite}$ $\cot A = \dfrac{b}{a}$

The Unit Circle - The Unit Circle is a circle centered at the origin with a radius of 1.
Theta θ - The letter used to represent the angle.
Reference Angle - The measure of the angle between the *x*-axis and the hypotenuse.



$\dfrac{\cos\theta}{1} = \dfrac{x}{1}$

$x = \cos\theta$

$\dfrac{\sin\theta}{1} = \dfrac{y}{1}$

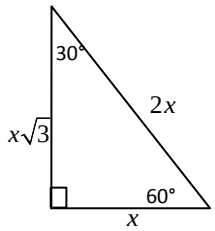
$y = \sin\theta$

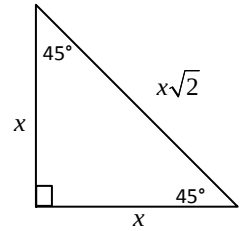
$(x,y) = (\cos\theta, \sin\theta)$

Special Right Triangles

30° - 60° - 90°

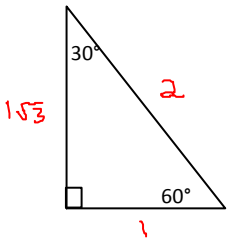
45° - 45° - 90°





$x = 1$

$x = 1$

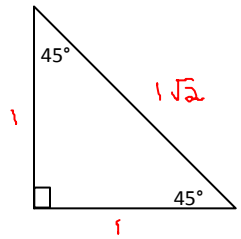


$\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{1}{2}$

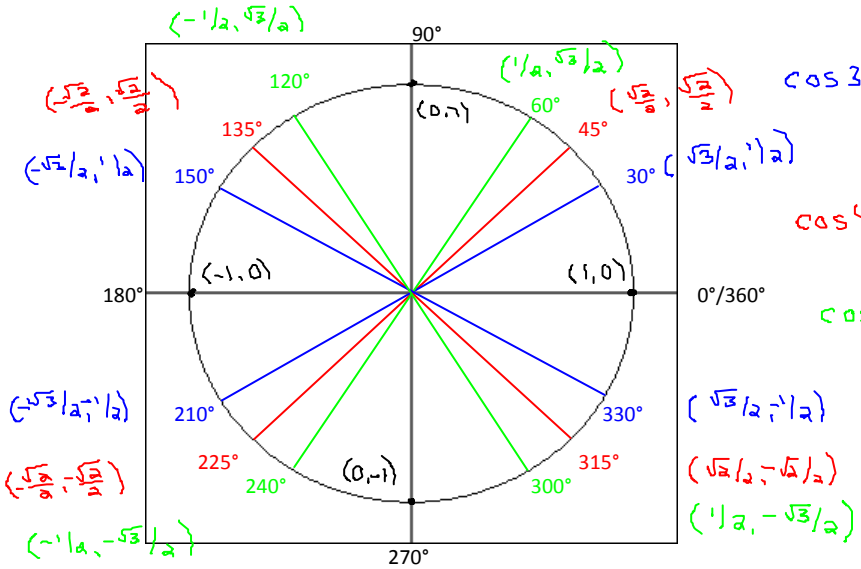
$\cos 60 = \frac{1}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$



$\cos 45 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin 45 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



$\cos 30 = \frac{\sqrt{3}}{2}$ $\sin 30 = \frac{1}{2}$

$\cos 45 = \frac{\sqrt{2}}{2}$ $\sin 45 = \frac{\sqrt{2}}{2}$

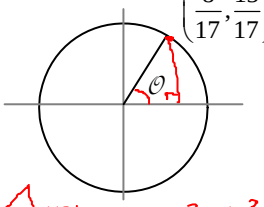
$\cos 60 = \frac{1}{2}$ $\sin 60 = \frac{\sqrt{3}}{2}$

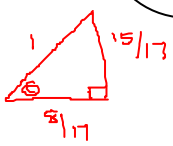
1. Determine the exact value of the six trigonometric functions of the angle $\mathcal{O}$.

a)

$\cos\theta$ $\sin\theta$

$\left(\frac{8}{17}, \frac{15}{17}\right)$





$a^2 + b^2 = c^2$

$\left(\frac{8}{17}\right)^2 + \left(\frac{15}{17}\right)^2 = c^2$

$c^2 = \frac{64}{289} + \frac{225}{289}$

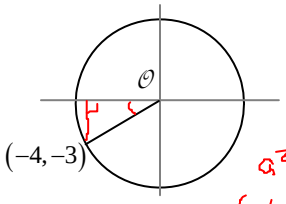
$c^2 = \frac{289}{289} = 1$ $c = 1$

$\sin\theta = \frac{15}{17}$ $\csc\theta = 17/15$

$\cos\theta = \frac{8}{17}$ $\sec\theta = 17/8$

$\tan\theta = 15/17 \; / \; \frac{15}{\;}$ $\cot\theta = 8/15$

b)



$a^2 + b^2 = c^2$

$(-4)^2 + (-3)^2 = c^2$

$16 + 9 = c^2$

$c^2 = 25$

$c = 5$

$\sin\theta = -3/5$ $\csc\theta = -5/3$

$\cos\theta = -4/5$ $\sec\theta = -5/4$

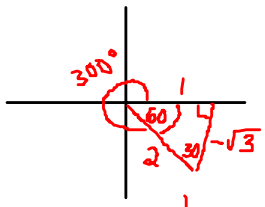
$\tan\theta = \frac{-3}{-4} = \frac{3}{\;}$ $\cot\theta = 4/3$

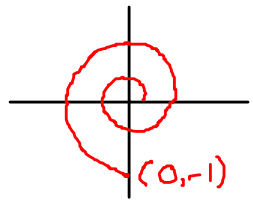
$\tan\theta = \frac{15}{17} \Big/ \frac{8}{17} = \frac{15}{8}$ $\quad \cot\theta = 8/15$ $\qquad \tan\theta = \frac{-3}{-4} = \frac{3}{4}$ $\quad \cot\theta = 4/3$

2. Find the point *(x, y)* that corresponds to the real number *t*.

a) $t = \frac{5\pi}{3} \cdot \frac{180}{\pi} = 300°$

b) $t = \frac{7\pi}{2} \cdot \frac{180}{\pi} = 630°$





$x = \cos 300° = \frac{1}{2}$

$y = \sin 300° = \frac{-\sqrt{3}}{2}$

$\boxed{(1/2, -\sqrt{3}/2)}$

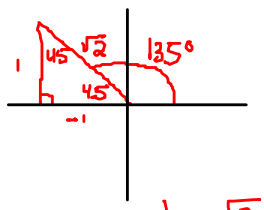
$x = \cos 630° = 0$

$y = \sin 630° = -1$

$\boxed{(0,-1)}$

3. Evaluate, if possible, the six trigonometric functions of the real number.

a) $t = \frac{3\pi}{4} \cdot \frac{180}{\pi} = 135°$



Reference $\measuredangle = 180° - 135° = 45°$

$\sin 135 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\csc 135 = \frac{\sqrt{2}}{1} = \sqrt{2}$

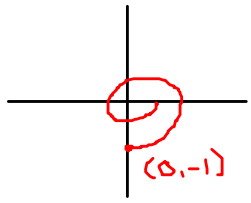
$\cos 135 = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\sec 135 = \frac{-\sqrt{2}}{1} = -\sqrt{2}$

$\tan 135 = \frac{1}{-1} = -1$

$\cot 135 = \frac{-1}{1} = -1$

b) $t = -\frac{5\pi}{2} \cdot \frac{180}{\pi} = -450°$



$\sin -450° = -1$

$\cos -450° = 0$

$\tan -450° = \dfrac{\sin -450°}{\cos -450°} = \dfrac{-1}{0} =$ undefined

$\csc -450° = \dfrac{1}{-1} = -1$

$\sec -450° = \dfrac{1}{0} =$ undefined

$\cot -450° = \dfrac{0}{-1} = 0$