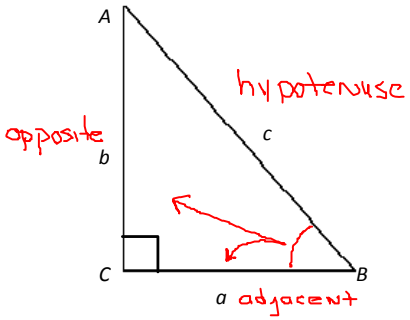
Right Triangle Trigonometry

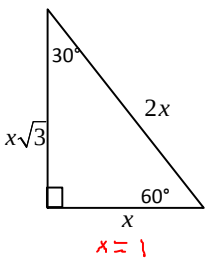


Definitions of Trigonometric Functions

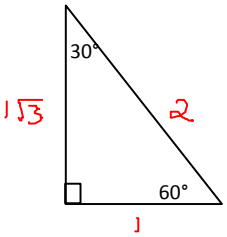
"sine"	$\sin\measuredangle = \frac{opposite}{hypotenuse}$	$\sin B = \frac{b}{c}$	"cosecant"	$\csc\measuredangle = \frac{hypotenuse}{opposite}$	$\csc B = \frac{c}{b}$
"cosine"	$\cos\measuredangle = \frac{adjacent}{hypotenuse}$	$\cos B = \frac{a}{c}$	"secant"	$\sec\measuredangle = \frac{hypotenuse}{adjacent}$	$\sec B = \frac{c}{a}$
"tangent"	$\tan\measuredangle = \frac{opposite}{adjacent}$	$\tan B = \frac{b}{a}$	"cotangent"	$\cot\measuredangle = \frac{adjacent}{opposite}$	$\cot B = \frac{a}{b}$

Special Right Triangles

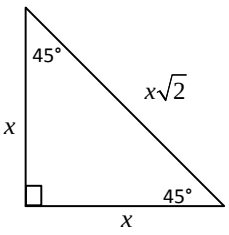
30° - 60° - 90°



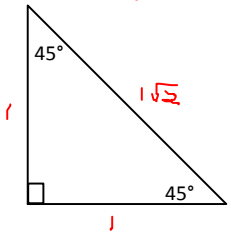
$x = 1$



45° - 45° - 90°

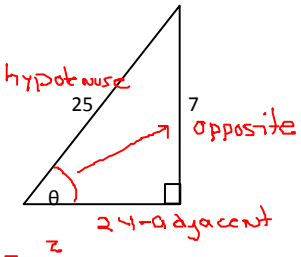


$x = 1$



1. Find the exact values of the six trigonometric functions of the angle θ.

a)



$a^2+b^2=c^2$

$a^2+7^2=25^2$

$a^2+49=625$

$-49 \quad -49$

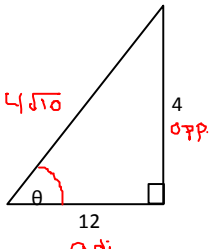
$a^2=576 \quad a=24$

$\sin\theta = 7/25 \qquad \csc\theta = 25/7$

$\cos\theta = 24/25 \qquad \sec\theta = 25/24$

$\tan\theta = 7/24 \qquad \cot\theta = 24/7$

b)



$a^2+b^2=c^2$

$4^2+12^2=c^2$

$16+144=c^2$

$c^2=160$

$c=\sqrt{160}$

$c=\sqrt{16}\cdot\sqrt{10}=4\sqrt{10}$

$\sin\theta = \frac{4}{4\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{4\sqrt{10}}{40} = \frac{\sqrt{10}}{10}$

$\cos\theta = \frac{12}{4\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{12\sqrt{10}}{40} = \frac{3\sqrt{10}}{10}$

$\tan\theta = \frac{4}{12} = \frac{1}{3}$

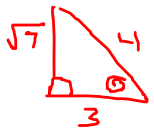
$\csc\theta = \frac{4\sqrt{10}}{4} = \sqrt{10}$

$\sec\theta = \frac{4\sqrt{10}}{12} = \frac{\sqrt{10}}{3}$

$\cot\theta = 3$

2. Find the exact values of the five remaining trigonometric functions of the angle θ.

a) $\cos\theta = \frac{3}{4}$ → adj / hyp



$a^2+b^2=c^2$

$3^2+b^2=4^2$

$9+b^2=16$

$-9 \quad -9$

$\sqrt{b^2}=\sqrt{7}$

$b=\sqrt{7}$

$\sin\theta = \frac{\sqrt{7}}{4}$

$\cos\theta = \frac{3}{4}$

$\tan\theta = \frac{\sqrt{7}}{3}$

$\csc\theta = \frac{4}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

$\sec\theta = 4/3$

$\cot\theta = \frac{3}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

b) $\sin\theta = \frac{\sqrt{2}}{4}$ → opp / hyp

$a^2+b^2=c^2$

$(\sqrt{2})^2+b^2=4^2$

$2+b^2=16$

$-2 \quad -2$

$\sqrt{b^2}=\sqrt{14}$

$\sin\theta = \frac{\sqrt{2}}{4}$

$\cos\theta = \frac{\sqrt{14}}{4}$ $\quad b = \sqrt{14}$

$\tan\theta = \frac{\sqrt{2}}{\sqrt{14}} = \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

$\csc\theta = \frac{4}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\sec\theta = \frac{4}{\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \frac{4\sqrt{14}}{14} = \frac{2\sqrt{14}}{7}$

$\cot\theta = \frac{\sqrt{7}}{1} = \sqrt{7}$

c) $\cot\theta = \frac{6}{1}$ $\quad \tan\theta = \frac{1}{6} \rightarrow \frac{opp}{adj}$

$a^2 + b^2 = c^2$

$1^2 + 6^2 = c^2$

$1 + 36 = c^2$

$\sqrt{c^2} = \sqrt{37}$

$c = \sqrt{37}$

$\sin\theta = \frac{1}{\sqrt{37}} \cdot \frac{\sqrt{37}}{\sqrt{37}} = \frac{\sqrt{37}}{37}$

$\cos\theta = \frac{6}{\sqrt{37}} \cdot \frac{\sqrt{37}}{\sqrt{37}} = \frac{6\sqrt{37}}{37}$

$\tan\theta = \frac{1}{6}$

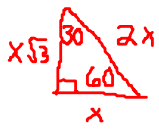
$\csc\theta = \frac{\sqrt{37}}{1} = \sqrt{37}$

$\sec\theta = \frac{\sqrt{37}}{6}$

$\cot\theta = 6$

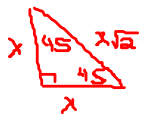
3. Evaluate each trigonometric function.

a) $\cos 60^\circ$



$\cos 60^\circ = \frac{1}{2}$

b) $\sin\frac{\pi}{4} \cdot \frac{180}{\pi} = 45^\circ$ (45)



$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

c) $\cot\frac{\pi}{3} \cdot \frac{180}{\pi} = 60^\circ$ (60)

$\tan 60^\circ = \frac{\sqrt{3}}{1}$

$\cot 60^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

4. Find the value of θ in degrees $\left(0^\circ < \theta < 90^\circ\right)$ and radians $\left(0 < \theta < \frac{\pi}{2}\right)$.

a) $\sin\theta = \frac{\sqrt{3}}{2}$ → OPP / → HYP

$\theta = 60^\circ$

$\theta = \frac{60 \cdot \pi}{180} = \frac{\pi}{3}$

b) $\cos\theta = \frac{\sqrt{2}}{2}$ → adj / → HYP

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = 45^\circ$

$\theta = \frac{45 \cdot \pi}{180} = \frac{\pi}{4}$

c) $\sec\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{3}{2\sqrt{3}}$

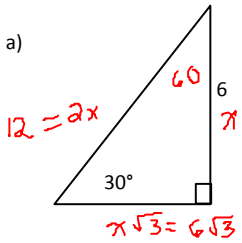
$\frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$

$\theta = \frac{30 \cdot \pi}{180_6} = \frac{\pi}{6}$

5. Find the missing sides of the triangle.

a)



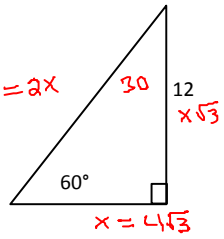
$12 = 2x$

$x\sqrt{3} = 6\sqrt{3}$

$x = 6$

$6\sqrt{3}, 12$

b)



$8\sqrt{3} = 2x$

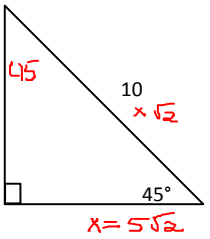
$x = 4\sqrt{3}$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{12}{\sqrt{3}}$

$x = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$4\sqrt{3}, 8\sqrt{3}$

c)



$5\sqrt{2} = x$

$x = 5\sqrt{2}$

$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{10}{\sqrt{2}}$

$x = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

$5\sqrt{2}, 5\sqrt{2}$