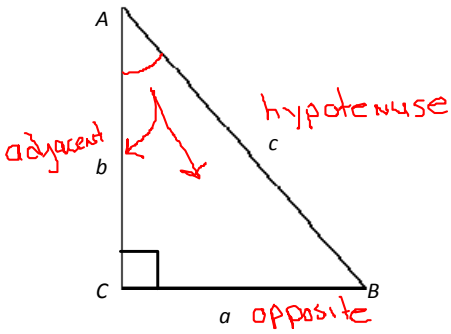
Trigonometric Functions of an Angle - Part 1



Definitions of Trigonometric Functions

"sine" $\sin\measuredangle = \dfrac{opposite}{hypotenuse}$ $\sin A = \dfrac{a}{c}$

"cosecant" $\csc\measuredangle = \dfrac{hypotenuse}{opposite}$ $\csc A = \dfrac{c}{a}$

"cosine" $\cos\measuredangle = \dfrac{adjacent}{hypotenuse}$ $\cos A = \dfrac{b}{c}$

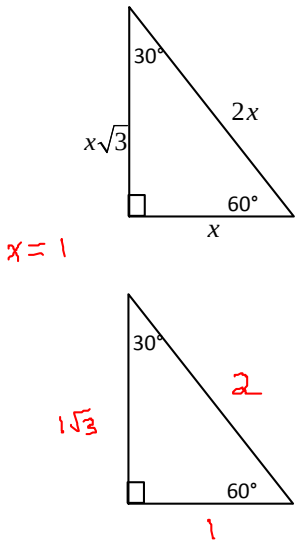
"secant" $\sec\measuredangle = \dfrac{hypotenuse}{adjacent}$ $\sec A = \dfrac{c}{b}$

"tangent" $\tan\measuredangle = \dfrac{opposite}{adjacent}$ $\tan A = \dfrac{a}{b}$

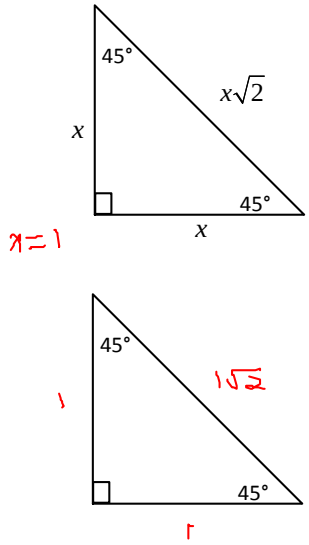
"cotangent" $\cot\measuredangle = \dfrac{adjacent}{opposite}$ $\cot A = \dfrac{b}{a}$

Special Right Triangles

30° - 60° - 90°

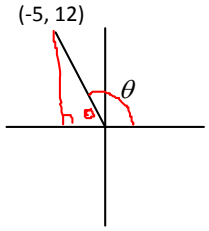


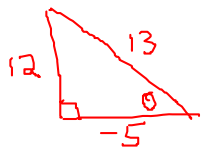
45° - 45° - 90°



1. Determine the exact values of the six trigonometric functions of the angle θ.

a)





$a^2+b^2=c^2$

$(-5)^2+(12)^2=c^2$

$25+144=c^2$

$169=c^2$

$c^2=169$

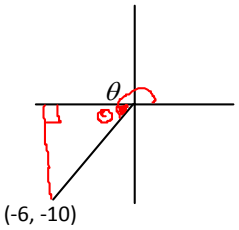
$c=13$

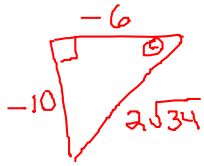
$\sin\theta=\frac{12}{13}$ $\csc\theta=\frac{13}{12}$

$\cos\theta=\frac{-5}{13}$ $\sec\theta=\frac{-13}{5}$

$\tan\theta=\frac{12}{-5}$ $\cot\theta=\frac{-5}{12}$

b)





$a^2+b^2=c^2$

$(-6)^2+(-10)^2=c^2$

$36+100=c^2$

$c^2=136$

$c=\sqrt{136}$

$c=\sqrt{4}\cdot\sqrt{34}$

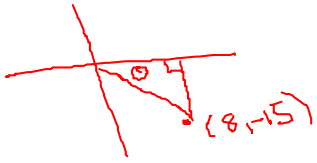
$c=2\sqrt{34}$

$\sin\theta=\frac{\overset{-5}{\cancel{-10}}}{\cancel{2}\sqrt{34}}=\frac{-5}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}}=\frac{-5\sqrt{34}}{34}$ $\csc\theta=\frac{\sqrt{34}}{-5}$

$\cos\theta=\frac{\overset{-3}{\cancel{-6}}}{\cancel{2}\sqrt{34}}=\frac{-3}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}}=\frac{-3\sqrt{34}}{34}$ $\sec\theta=\frac{\sqrt{34}}{-3}$

$\tan\theta=\frac{-10}{-6}=\frac{5}{3}$ $\cot\theta=\frac{3}{5}$

c) (8, -15)



$a^2+b^2=c^2$

$8^2+(-15)^2=c^2$

$64+225=c^2$

$c^2=289$

$c=17$

$\sin\theta=\frac{-15}{17}$ $\csc\theta=-\frac{17}{15}$

$\cos\theta=\frac{8}{17}$ $\sec\theta=\frac{17}{8}$

$\cos\theta = \frac{8}{17}$ $\qquad$ $\sec\theta = \frac{17}{8}$

$\tan\theta = \frac{-15}{8}$ $\qquad$ $\cot\theta = -\frac{8}{15}$

2. State the quadrant in which angle θ lies.

a) $\sin\theta > 0$ I, II
$\sec\theta < 0$ II, III

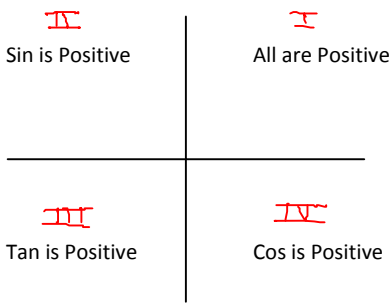
$\boxed{\text{II}}$

b) $\cot\theta > 0$ I, III
$\csc\theta > 0$ I, II

$\boxed{\text{I}}$

c) $\tan\theta > 0$ I, III
$\cos\theta < 0$ II, III

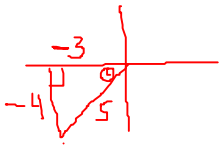
$\boxed{\text{III}}$



3. Find the values of the six trigonometric functions of angle θ.

a) $\cos\theta = -\frac{3}{5}$

θ is in Quadrant III.



$a^2 + b^2 = c^2$
$(-3)^2 + b^2 = 5^2$
$9 + b^2 = 25$
$b^2 = 16$
$b = 4$

$\sin\theta = -\frac{4}{5}$

$\cos\theta = -\frac{3}{5}$

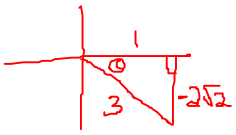
$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$

$\csc\theta = -\frac{5}{4}$

$\sec\theta = -\frac{5}{3}$

$\cot\theta = \frac{3}{4}$

b) $\sec\theta = 3$ $\qquad$ $\cos\theta = \frac{1}{3}$ I, IV
$\tan\theta < 0$ II, IV



$a^2 + b^2 = c^2$
$1^2 + b^2 = 3^2$
$1 + b^2 = 9$
$b^2 = 8$
$b = \sqrt{8}$
$b = \sqrt{4}\cdot\sqrt{2} = 2\sqrt{2}$

$\sin\theta = -\frac{2\sqrt{2}}{3}$

$\cos\theta = \frac{1}{3}$

$\tan\theta = -2\sqrt{2}$

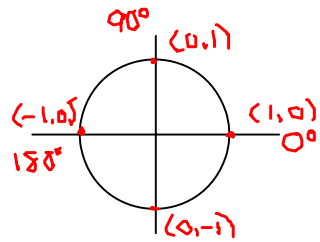
$\csc\theta = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{-4}$

$\sec\theta = 3$

$\cot\theta = \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{-4}$

c) $\tan\theta$ is undefined

$0 \le \theta \le \pi$



$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\tan 0° = \dfrac{\sin 0°}{\cos 0°} = \dfrac{0}{1} = 0$

$\tan 90° = \dfrac{\sin 90°}{\cos 90°} = \dfrac{1}{0}$ undefined $\boxed{\theta = 90°}$

$\tan 180° = \dfrac{\sin 180°}{\cos 180°} = \dfrac{0}{-1} = 0$

4. The terminal side of angle θ lies on the given line in the specified quadrant. Find the values of the six trigonometric functions of angle θ.

$y = \dfrac{1}{2}x$

Quadrant III

$m = \frac{1}{2}$ ↑1 →2

$b = (0,0)$

$a^2 + b^2 = c^2$

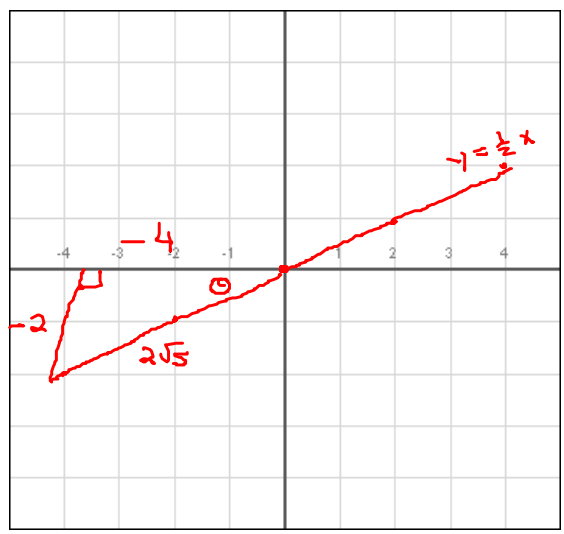
$(-4)^2 + (-2)^2 = c^2$

$16 + 4 = c^2$

$\sqrt{c^2} = \sqrt{20}$

$c = \sqrt{4} \cdot \sqrt{5}$

$c = 2\sqrt{5}$



$\sin\theta = \dfrac{-2}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{-\cancel{2}^{-1}\sqrt{5}}{\cancel{10}_{5}} = \dfrac{-\sqrt{5}}{5}$

$\cos\theta = \dfrac{-4}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{-\cancel{4}^{-2}\sqrt{5}}{\cancel{10}_{5}} = \dfrac{-2\sqrt{5}}{5}$

$\tan\theta = \dfrac{-2}{-4} = \dfrac{1}{2}$

$\csc\theta = \dfrac{\cancel{2}\sqrt{5}}{-\cancel{2}} = -\sqrt{5}$

$\sec\theta = \dfrac{\cancel{2}^{1}\sqrt{5}}{\cancel{-4}_{-2}} = \dfrac{-\sqrt{5}}{2}$

$\cot\theta = \dfrac{2}{1} = 2$