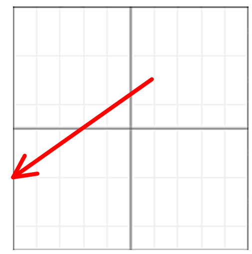
Vectors in the Plane

Vector: A directed line segment that has a magnitude and a direction.



$x_1 \; y_1$

Initial Point: $P(1,2)$

$x_2 \; y_2$

Terminal Point: $Q(-5,-2)$

Component Form of a Vector: v = $\langle x_2 - x_1, y_2 - y_1 \rangle$

$v = \langle -5-1, -2-2 \rangle$

$v = \langle -6, -4 \rangle$ $\quad \overrightarrow{PQ} = \langle -6, -4 \rangle$

Linear Combination of Vectors/Standard Unit Form of a Vector: v = xi + yj

$v = -6i + -4j$

Magnitude of a vector v: $\|v\| = \sqrt{x^2 + y^2}$

$\|v\| = \sqrt{(-6)^2 + (-4^2)}$

$\|v\| = \sqrt{36+16} = \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$

1. Let u = $\langle -3, 6 \rangle$ and v = $\langle 5, -7 \rangle$. Find each of the following vector operations.

a) 2v

$2v = \langle 2(5), 2(-7) \rangle$

$\boxed{2v = \langle 10, -14 \rangle}$

b) 3u − 4v

$3u = \langle 3(-3), 3(6) \rangle$

$3u = \langle -9, 18 \rangle$

$4v = \langle 4(5), 4(-7) \rangle$

$4v = \langle 20, -28 \rangle$

$3u - 4v = \langle -9, 18 \rangle - \langle 20, -28 \rangle$

$3u - 4v = \langle -9-20, 18 - -28 \rangle$

$\boxed{3u - 4v = \langle -29, 46 \rangle}$

2. Find a unit vector in the direction of $v = \langle -3, 2 \rangle$.

unit vector $= \dfrac{v}{\|v\|}$

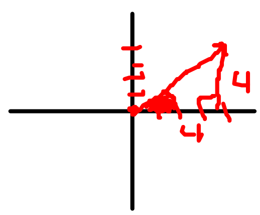
$\|v\| = \sqrt{(-3)^2+(2)^2}$
$= \sqrt{9+4}$
$= \sqrt{13}$

$\boxed{v = \dfrac{-3}{\sqrt{13}} i + \dfrac{2}{\sqrt{13}} j}$

3. Find the direction angle of each vector.

Direction Angle: $\tan\theta = \dfrac{y}{x}$

a) u = 4i + 4j $\langle 4, 4 \rangle$



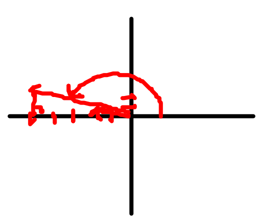
$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{4}{4}$

$\tan\theta = 1$

$\boxed{\theta = 45°}$

b) $v = -5i + 2j$ $v = \langle -5, 2 \rangle$



$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{2}{5}$

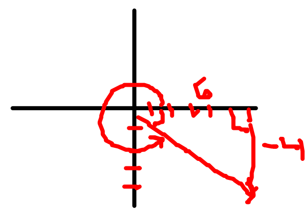
$\theta = 22°$

$180 - 22 = \boxed{158°}$

4. Find the vector v with a magnitude of 5 in the same direction as u = 6i − 4j.

$u = \langle 6, -4 \rangle$

v = $\|v\| \cos\theta$i + $\|v\| \sin\theta$j



$\|v\| = 5$

θ (direction $\measuredangle$) $= 326°$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{4}{6}$

$\theta = 34°$

$360 - 34 = 326°$

$v = \underbrace{5\cos 326 i}_{4.15} + \underbrace{5\sin 326 j}_{-2.80}$

$\boxed{v = 4.15i - 2.80j}$

5. Find the magnitude and direction angle of vector v.

a) $v = 4(\cos 225° i + \sin 225° j)$

$V = 4\cos 225° i + 4\sin 225° j$

$\|v\|$ θ $\|v\|$ θ

$\boxed{\begin{array}{l} \|v\| = 4 \\ \theta = 225° \end{array}}$

b) v = −3i + 4j

$\|v\| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{4}{3}$

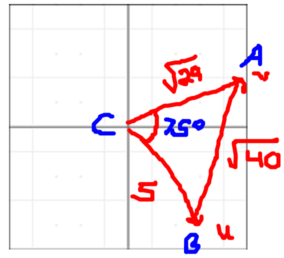
$\theta = 53°$

Direction $\measuredangle = 180 - 53 = 127°$

$\boxed{\begin{array}{l} \|v\| = 5 \\ \theta = 127° \end{array}}$

6. Use the law of cosines to find the angle between the given vectors.

u = 3i − 4j
v = 5i + 2j



$\|u\| = \sqrt{(3)^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\|v\| = \sqrt{(5)^2+(2)^2} = \sqrt{25+4} = \sqrt{29}$

$u - v = \langle 3-5,\ -4-2 \rangle = \langle -2, -6 \rangle$

$\|u-v\| = \sqrt{(-2)^2+(-6)^2} = \sqrt{4+36} = \sqrt{40}$

$c^2 = a^2 + b^2 - 2ab\cdot\cos C$

$\underbrace{(\sqrt{40})^2} = \underbrace{(5)^2 + (\sqrt{29})^2} - \underbrace{2(5)(\sqrt{29})}\cos C$

$40 = 54 - 10\sqrt{29}\cos C$

$-54 \quad -54$

$\dfrac{-14}{-10\sqrt{29}} = \dfrac{-10\sqrt{29}\cos C}{-10\sqrt{29}}$

$\cos C = .2599$

$\boxed{\measuredangle C = 75^\circ}$