Trigonometric Form of a Complex Number

Complex Form to Trigonometric Form

$x + yi \to r\cos\theta + r\sin\theta i$

$(x, y) \to (r, \theta)$

$$\boxed{r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}}$$

Trigonometric Form to Complex Form

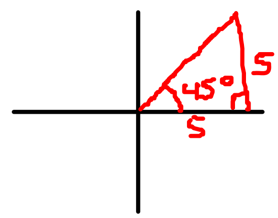
$r\cos\theta + r\sin\theta i \to x + yi$

$(r, \theta) \to (x, y)$

$$\boxed{x = r\cos\theta \qquad y = r\sin\theta}$$

1. Write the complex number in trigonometric form.

a) $5+5i$ $\quad x=5 \quad y=5$



$R: \sqrt{(5)^2+(5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$\tan\theta = \frac{5}{5}$

$\tan\theta = 1 \qquad \theta = 45^\circ$

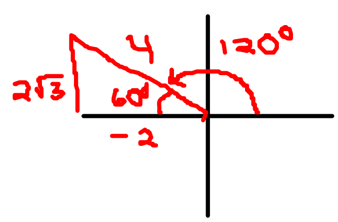
$5\sqrt{2}\cos 45^\circ + 5\sqrt{2}\sin 45^\circ i$

$5\sqrt{2}(\cos \pi/4 + \sin \pi/4 i)$

$5\sqrt{2} \operatorname{cis} \pi/4$

b) $-2(1-\sqrt{3}i) = -2 + 2\sqrt{3}i$

$x = -2 \quad y = 2\sqrt{3}$



$R = \sqrt{(-2)^2 + (2\sqrt{3})^2}$

$= \sqrt{4 + 4\cdot 3}$

$= \sqrt{4+12}$

$= \sqrt{16}$

$= 4$

$\tan\theta = \frac{2\sqrt{3}}{-2}$

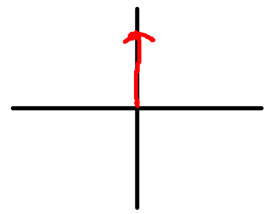
$= -\frac{\sqrt{3}}{1}$

$4(\cos 120^\circ + \sin 120^\circ i)$

$4\cos 2\pi/3 + 4\sin 2\pi/3 i$

$4 \operatorname{cis} 2\pi/3$

c) 7i $x = 0$ $y = 7$



$R = \sqrt{0^2 + 7^2} = \sqrt{49} = 7$

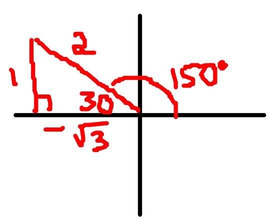
$\tan\theta = \frac{7}{0}$

$\theta = 90°$

$\boxed{7(\cos 90° + \sin 90° i)}$

2. Write the complex number in standard form.

a) $3\cos(150^\circ) + 3\sin(150^\circ)\,i$ $R = 3$
$\theta = 150°$

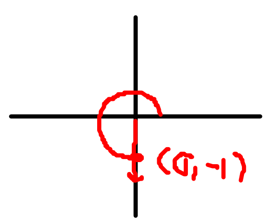


$x = R\cos\theta$
$= 3\cos 150°$
$= 3\left(-\frac{\sqrt{3}}{2}\right)$
$= \frac{-3\sqrt{3}}{2}$

$y = R\sin\theta$
$= 3\sin 150°$
$= 3\left(\frac{1}{2}\right)$
$= \frac{3}{2}$

$\boxed{\frac{-3\sqrt{3}}{2} + \frac{3}{2}i}$

b) $9\cos\frac{3\pi}{2} + 9\sin\frac{3\pi}{2}$



$R = 9$ $\theta = \frac{3\pi}{2}$ or $270°$

$x = R\cos\theta$
$= 9\cos 270°$
$= 9(0)$
$= 0$

$y = R\sin\theta$
$= 9\sin 270°$
$= 9(-1)$
$= -9$

$0-9i = \boxed{-9i}$

Multiplication and Division of Complex Numbers

$$z_1 = r_1(\cos\theta_1 + \sin\theta_1 i)$$
$$z_2 = r_2(\cos\theta_2 + \sin\theta_2 i)$$

$$z_1 \bullet z_2 = r_1 \bullet r_2 \left[\cos(\theta_1+\theta_2) + \sin(\theta_1+\theta_2)i\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + \sin(\theta_1-\theta_2)i\right]$$

3. Perform the operation.

a) $\left[3\left(\cos\frac{4\pi}{3} + \sin\frac{4\pi}{3}i\right)\right] \bullet \left[\frac{1}{2}\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}i\right)\right]$

$\left(3 \operatorname{cis} \frac{4\pi}{3}\right) \cdot \left(\frac{1}{2} \operatorname{cis} \frac{\pi}{4}\right)$

$3 \cdot \frac{1}{2} \operatorname{cis} \left(\frac{4\pi \cdot 4}{3 \cdot 4} + \frac{\pi \cdot 3}{4 \cdot 3}\right)$ LCD = 12 $\quad \frac{16\pi}{12} + \frac{3\pi}{12} = \frac{19\pi}{12}$

$\boxed{\frac{3}{2} \operatorname{cis}\left(\frac{19\pi}{12}\right) \qquad \frac{3}{2}\left(\cos\frac{19\pi}{12} + \sin\frac{19\pi}{12} i\right)}$

b) $\dfrac{32\left(\cos 150^\circ + \sin 150^\circ i\right)}{24\left(\cos 20^\circ + \sin 20^\circ i\right)} = \dfrac{32 \text{ cis } 150^\circ}{24 \text{ cis } 20^\circ}$

$\dfrac{32 \div 8}{24 \div 8} = \dfrac{4}{3}$

$\dfrac{4}{3} \text{ cis } (150-20) = \boxed{\dfrac{4}{3} \text{ cis } 130^\circ}$

$\boxed{= \dfrac{4}{3}\left(\cos 130^\circ + \sin 130^\circ i\right)}$

De Moivre's Theorem - Used to raise a complex number to the n^{th} power and to the n^{th} root

→ n^{th} power

$$\left[r\left(\cos\theta + \sin\theta i\right)\right]^n = r^n\left(\cos n \bullet \theta + \sin n \bullet \theta i\right)$$

4. Use De Moivre's Theorem to find the indicated power of the complex number.

a) $\left[2\left(\cos 40^\circ + \sin 40^\circ i\right)\right]^6$ $\quad (2 \text{ cis } 40)^6$

$2^6 \text{ cis}(40^\circ \cdot 6)$

$\boxed{64 \text{ cis } 240^\circ}$

b) $\left(1-\sqrt{3}i\right)^8$

$x=1 \quad y=-\sqrt{3}$

$R=\sqrt{(1)^2+(-\sqrt{3})^2}$

$=\sqrt{1+3}$

$=\sqrt{4}$

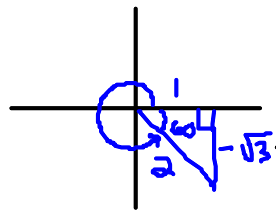
$=2$

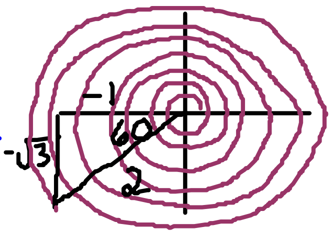
$\tan\theta=\frac{y}{x}$

$=\frac{-\sqrt{3}}{1}$

$\theta=300°$

$2\text{cis}\,300°$





$\left(2\text{ cis } 300°\right)^8$

$2^8\text{ cis } 300°\cdot 8$

$256\text{ cis } 2400°$

$x=R\cos\theta \quad y=R\sin\theta$

$x=256\cos 2400 \quad y=256\sin 2400$

$x=256\left(-\frac{1}{2}\right) \quad y=256\left(\frac{-\sqrt{3}}{2}\right)$

$x=-128$

$y=-128\sqrt{3}$

$\boxed{-128-128\sqrt{3}i}$

n^{th} root

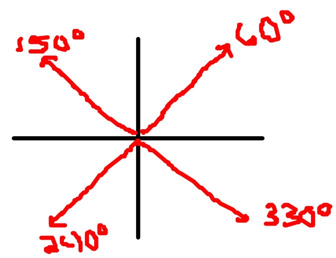
$$\sqrt[n]{\left[r(\cos\theta+\sin\theta i)\right]}$$

1) 1^{st} Root = $\dfrac{\theta}{n}$

2) All other roots are $\dfrac{360^\circ}{n}$ apart

5. Use De Moivre's Theorem to find the indicated root of the complex number.

a) 4^{th} roots of $z=16\left(\cos\dfrac{4\pi}{3}+\sin\dfrac{4\pi}{3}i\right)$



$N=4$ $\quad 16 \text{ cis } \frac{4\pi}{3}$

1^{st} root $= \dfrac{\theta}{N} = \dfrac{\frac{4\pi}{3}}{4} = \dfrac{4\pi}{3}\cdot\dfrac{1}{4} = \dfrac{\pi}{3}$

$= 60^\circ$

Other roots $= \dfrac{360}{N} = \dfrac{360}{4} = 90^\circ$

$60^\circ = \frac{\pi}{3}$
$150^\circ = \frac{5\pi}{6}$
$240^\circ = \frac{4\pi}{3}$
$330^\circ = \frac{11\pi}{6}$

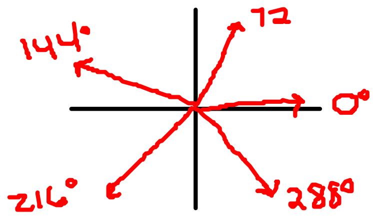
$\sqrt[4]{16} = 2$

$$\boxed{\begin{array}{l} 2 \text{ cis } \pi/3 \\ 2 \text{ cis } 5\pi/6 \\ 2 \text{ cis } 4\pi/3 \\ 2 \text{ cis } 11\pi/6 \end{array}}$$

b) 5^{th} roots of 1

$1 + 0i$

$N = 5 \quad \theta = 0^\circ \qquad 1 \text{ cis } 0^\circ$



1^{st} root $= \dfrac{\theta}{N} = \dfrac{0}{5} = 0^\circ$

$\dfrac{360}{N} = \dfrac{360}{5} = 72^\circ$

$\sqrt[5]{1} = 1$

$$\boxed{\begin{array}{l} 1 \text{ cis } 0^\circ \\ 1 \text{ cis } 72^\circ \\ 1 \text{ cis } 144^\circ \\ 1 \text{ cis } 216^\circ \\ 1 \text{ cis } 288^\circ \end{array}}$$

c) cube roots of $2-2i$

$x = 2 \quad y = -2$

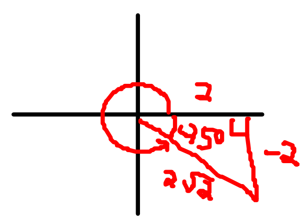
$N = 3$

$R = \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

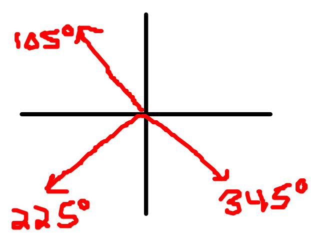


$\tan\theta = \dfrac{y}{x}$

$= \dfrac{-2}{2}$

$= -1$

$\theta = 315^\circ$



$2\sqrt{2} \text{ cis } 315^\circ$

1^{st} root $\quad \dfrac{\theta}{N} = \dfrac{315^\circ}{3} = 105^\circ$

$\dfrac{360}{N} = \dfrac{360}{3} = 120$

$$\boxed{\begin{array}{l} \sqrt[3]{2\sqrt{2}} \text{ cis } 105^\circ \\ \sqrt[3]{2\sqrt{2}} \text{ cis } 225^\circ \\ \sqrt[3]{2\sqrt{2}} \text{ cis } 345^\circ \end{array}}$$

$\frac{360}{N} = \frac{360}{3} = 120$

$\boxed{\sqrt[3]{2\sqrt{2}}\ \operatorname{cis} 345^\circ}$