The Natural Logarithmic Function

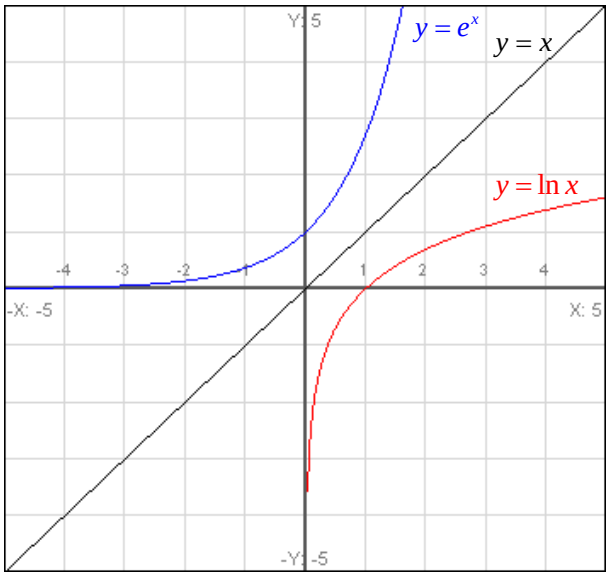
The irrational number $e \approx 2.71828...$ is called the natural base. It represents the number whose natural log is 1.

Natural Exponential Function

$y = e^x$

Natural Logarithmic Function

$y = \log_e x$ $\quad y = \ln x$



Properties of the Natural Exponential Function
Domain: All Real Numbers
Range: *y > 0*
y-intercept: *(0, 1)*
Contains the point (1, *e*)
Horizontal Asymptote: *y = 0*
The function is increasing.

Properties of the Natural Logarithmic Function
Domain: *x > 0*
Range: All Real Numbers
x-intercept: *(1, 0)*
Contains the point (*e*, 1)
Vertical Asymptote: *x = 0*
The function is increasing.

Properties of Natural Logarithms

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$
5. If $\ln x = \ln y$, then $x = y$.
6. $\ln(u \cdot v) = \ln u + \ln v$
7. $\ln \frac{u}{v} = \ln u - \ln v$
8. $\ln u^n = n \cdot \ln u$
9. $\ln \sqrt[n]{u} = \frac{1}{n} \cdot \ln u$

Directions: Rewrite each expression in exponential form.

1. $\ln 4 = 1.386$
2. $\ln 1 = 0$

Directions: Solve for *x* using the properties of logarithms.

3. $\ln e^7 = x$

4. $\ln 1 = \ln x$

Directions: Use the properties to expand each natural logarithm.

5. $\ln \sqrt{\frac{x^2 y}{z}}$

6. $\ln \frac{x^3}{yz^4}$

7. $\ln \sqrt{x\left(x^2 - 2\right)}$

Directions: Write each expression as a single logarithm.

8. $\frac{1}{2}\left[\ln(x+2)-3\ln x\right]$

9. $\frac{3}{2}\ln 4x^2-\frac{2}{3}\ln x^{15}$

Directions: Solve for *x* in each exponential equation. Round your answer to three decimal places.

10. $500e^{-x}=300$

11. $-3+4e^{2x}=5$

Directions: Solve for *x* in each logarithmic equation. Round your answer to three decimal places.

12. $\ln x - \ln 5 = \ln 3$

13. $\ln(2x+5) = 10$

14. $\ln \sqrt{x+3} = 3$

15. $\ln(y+6) - \ln y = \ln(y+2)$