Continuity

<u>A function is continuous at *x* = *a* if:</u>

$a)\ f(a)$ exists.

$b)\ \lim_{x \to a} f(x)$ exists.

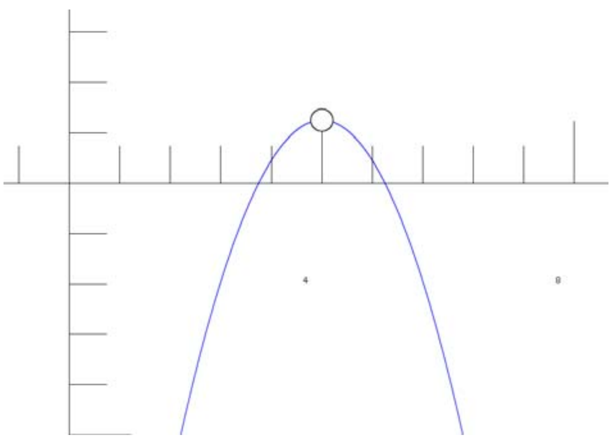
$c)\ \lim_{x \to a} f(x) = f(a)$

<u>A function is not continuous at:</u>

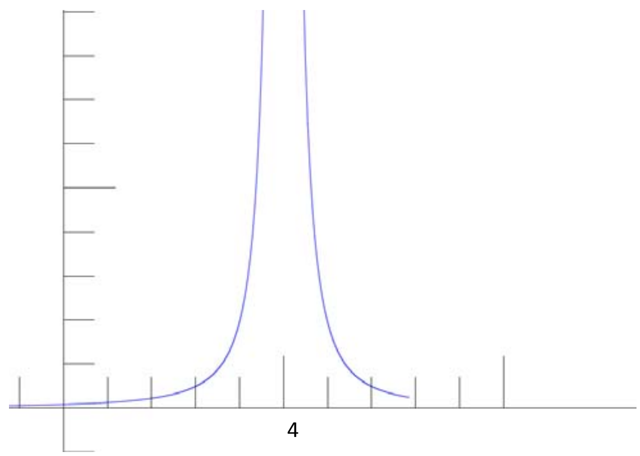
a) Vertical Asymptotes
b) Deleted Points/Holes
c) Breaking Points

<u>Examples of Discontinuous Functions:</u>

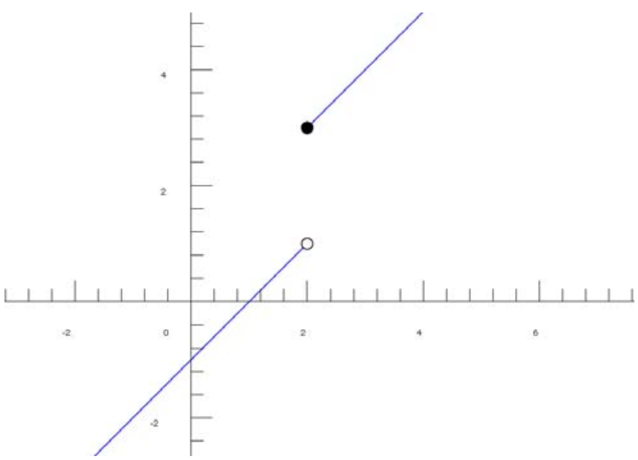
Deleted Point/Hole at *x* = 4



Asymptote at *x* = 4

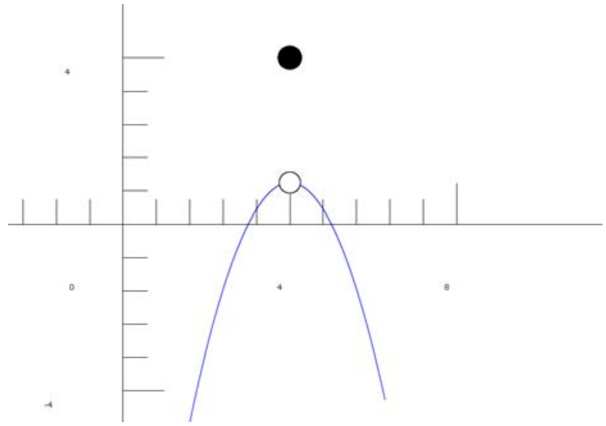


Breaking Point at *x* = 2



Discontinuous at x = 4 because

$$\lim_{x\to 4} f(x) \neq f(4)$$



1. Determine if the function is continuous at x = 2.

$$a)\ f(x) = \frac{1}{x-2}$$

$$b)\ f(x) = \begin{cases} 3x^2 - 1 & \text{for} \quad x < 2 \\ 2x + 5 & \text{for} \quad x \geq 2 \end{cases}$$

$$c)\ f(x) = \begin{cases} \dfrac{x^2 - 4}{x-2} & \text{if} \quad x \neq 2 \\ 5 & \text{if} \quad x = 2 \end{cases}$$

2. Find *c* if *f(x)*is continuous at *x* = 2.

$$f(x)=\begin{cases} x+3 & \text{for } x\le 2 \\ cx+6 & \text{for } x>2 \end{cases}$$

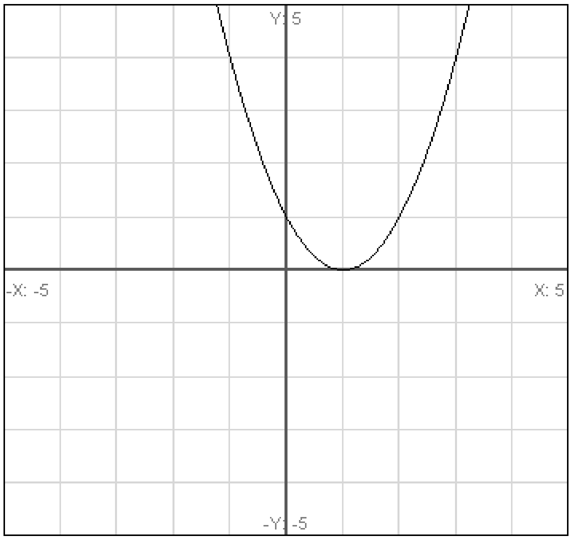
3. Find *c* if *f(x)* is continuous at *x* = 3.

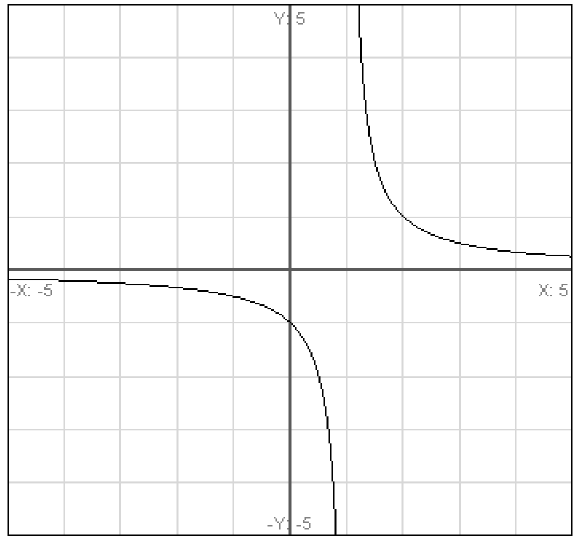
$$f(x)=\begin{cases} \dfrac{x^3-27}{x-3} & \text{if } x\ne 3 \\ c & \text{if } x=3 \end{cases}$$

4. Determine the points where the function is discontinuous.

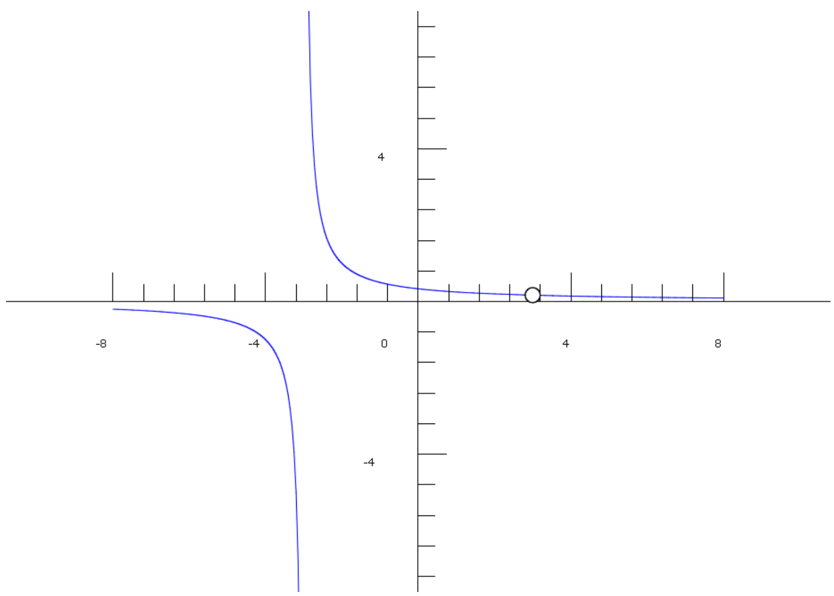
a) $f(x) = x^2 - 2x + 1$

b) $f(x) = \dfrac{1}{x-1}$

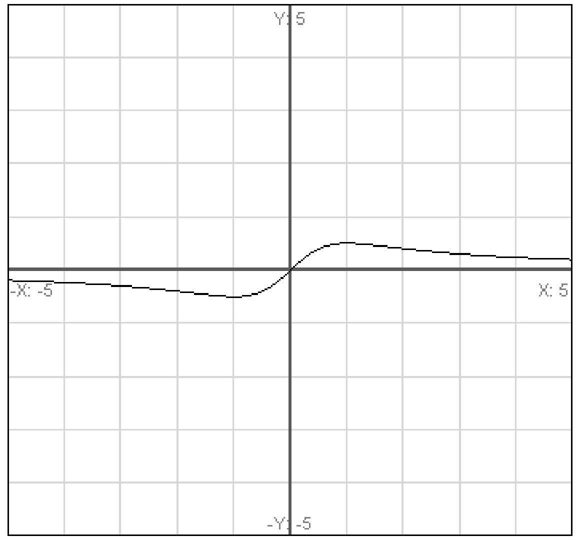




c) $f(x) = \dfrac{x-3}{x^2-9}$



d) $f(x)=\dfrac{x}{x^2+1}$



e) $f(x)=\begin{cases}3+x^2 & \text{for } x\le 2\\ x^2+1 & \text{for } x>2\end{cases}$