Relationship Between Continuity and Differentiability

<u>A function is continuous at $x = a$ if:</u>

a) $f(a)$ exists

b) $\lim\limits_{x \to a} f(x)$ exists

c) $\lim\limits_{x \to a} f(x) = f(a)$

<u>A function is differentiable at $x = a$ if:</u>

a) $f(a)$ is continuous

b) $f'_-(a) = f'_+(a)$

<u>Geometric conditions that prevent a function from being differentiable at a point :</u>

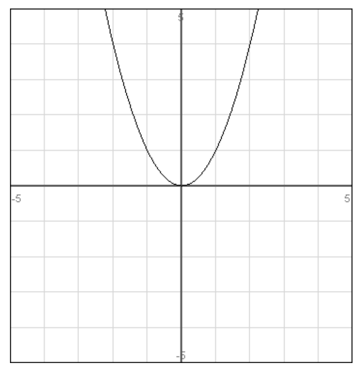
a) Any point of discontinuity (asymptote, deleted point)

b) Corner point

c) Cusp

<u>A function that is continuous and differentiable.</u>

$f(x) = x^2$



$\lim\limits_{x \to 0^-} x^2 =$

$\lim\limits_{x \to 0^+} x^2 =$

$\lim\limits_{x \to 0} x^2 =$

$f(0) =$

$f'(x) =$

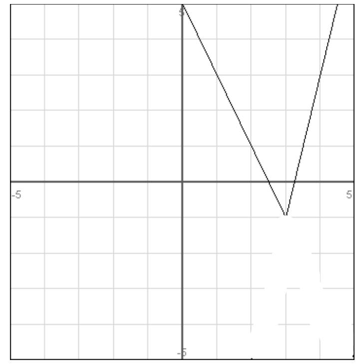
$f'_-(0) =$

$f'_+(0) =$

$f'(0) =$

<u>A function that is continuous but not differentiable.</u>

$$f(x) = \begin{cases} 5 - 2x & \text{for } x < 3 \\ 4x - 13 & \text{for } x \geq 3 \end{cases}$$



$\lim_{x \to 3^-} f(x) =$

$\lim_{x \to 3^+} f(x) =$

$\lim_{x \to 3} f(x) =$

$f(3) =$

$f'(x) = \begin{cases} \\ \\ \end{cases}$

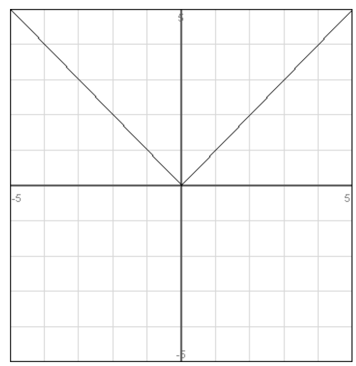
$f'_{-}(3) =$

$f'_{+}(3) =$

$f'(3) =$

<u>A function that is continuous but not differentiable.</u>

$f(x) = |x|$



$\lim_{x \to 0^-} |x| =$

$\lim_{x \to 0^+} |x| =$

$\lim_{x \to 0} |x| =$

$f(0) =$

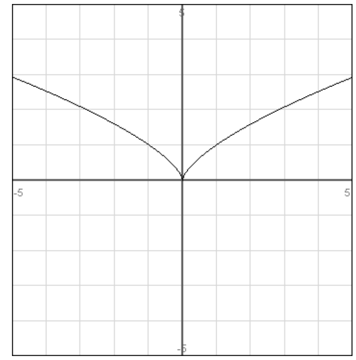
$f'(x) = \begin{cases} \\ \\ \end{cases}$

$f'_{-}(0) =$

$f'_{+}(0) =$

$f'(0) =$

A function that is continuous but not differentiable.



$$f(x) = x^{\frac{2}{3}}$$

$$\lim_{x \to 0^-} x^{\frac{2}{3}} =$$

$$\lim_{x \to 0^+} x^{\frac{2}{3}} =$$

$$\lim_{x \to 0} x^{\frac{2}{3}} =$$

$$f(0) =$$

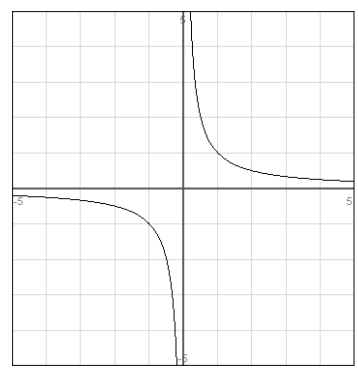
$$f'(x) =$$

$$f'_-(0) =$$

$$f'_+(0) =$$

$$f'(0) =$$

A function that is not continuous and therefore not differentiable.



$$f(x) = \frac{1}{x}$$

$$\lim_{x \to 0^-} \frac{1}{x} =$$

$$\lim_{x \to 0^+} \frac{1}{x} =$$

$$\lim_{x \to 0} \frac{1}{x} =$$

$$f(0) =$$

$$f'(x) =$$

$$f'_-(0) =$$

$$f'_+(0) =$$

$$f'(0) =$$

If a function is differentiable at a point, then it is continuous at that point.

If a function is not continuous at a point, then it is not differentiable at that point.

1. $f'(1)$ exists. Find the values of a and b.

$$f(x)=\begin{cases} x^2 & \text{for } x<1 \\ ax+b & \text{for } x\geq 1 \end{cases}$$

2. $f(x)$ is continuous and differentiable. Find the values of a and b.

$$f(x)=\begin{cases} ax^3-4x & \text{for } x\leq 1 \\ bx^2+2 & \text{for } x>1 \end{cases}$$

3. Determine if $f(x)$ is continuous and differentiable at $x = 2$.

$$f(x) = \begin{cases} x^2 + 4 & \text{for } x < 2 \\ 3x + 2 & \text{for } x \geq 2 \end{cases}$$