# Real Zeros of Polynomial Functions (Remainder and Factor Theorems, Rational Root Test and Upper/Lower Bounds)

Remainder Theorem - If a polynomial function $f(x)$ is divided by $x-k$, the remainder $f(k)=r$.

1. Use the Remainder Theorem to evaluate $f(2)$ in $f(x)=2x^3-3x^2+8x-7$.

Factor Theorem - A polynomial function $f(x)$ has a factor $x-k$ if and only if $f(k)=0$.

2. Determine if $x-2$ is a factor of $f(x)=6x^4-11x^3-11x^2+14x+8$.

Rational Zero Test - If a polynomial function has integer coefficients then the possible rational zeros are of the form $\frac{p}{q}$, where $q$ represents the factors of the leading coefficient and $p$ represents the factors of the constant.

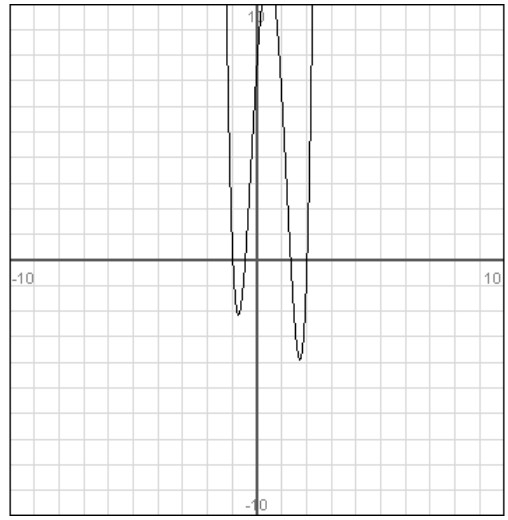
3. List the possible rational zeros of $f(x)$.

a) $f(x)=x^3+3x^2+x+6$

b) $f(x)=2x^4-3x^2+12$

4. Find all the real zeros of each polynomial equation.

a) $f(x) = 6x^4 - 11x^3 - 11x^2 + 14x + 8$



b) $f\left(x\right) = 2x^4 - 19x^3 + 42x^2 + 9x - 54$

Upper and Lower Bound Rule - If a polynomial function has real coefficients with a positive leading coefficient, when the polynomial is divided by $x - c$ using synthetic division,
If $c > 0$ and the last row is either positive or zero, then $c$ is an upper bound.
If $c < 0$ and the last row alternates from positive to negative, then $c$ is a lower bound.

5. Use synthetic division to determine if each value of $x$ is an upper bound or a lower bound.

$$f(x) = x^4 - 4x^3 + 16x - 16$$

a) $x = 5$

b) $x = -3$