Trigonometric Form of a Complex Number

<u>Complex Form to Trigonometric Form</u>

$$x + y\text{i} \;\rightarrow\; r\cos\theta + r\sin\theta\text{i}$$
$$(x, y) \;\rightarrow\; (r, \theta)$$

$$r = \sqrt{x^2 + y^2}$$
$$\tan\theta = \frac{y}{x}$$

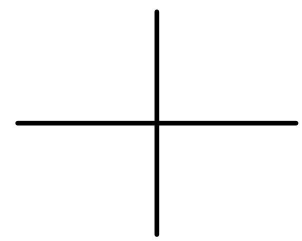
<u>Trigonometric Form to Complex Form</u>

$$r\cos\theta + r\sin\theta\text{i} \;\rightarrow\; x + y\text{i}$$
$$(r, \theta) \;\rightarrow\; (x, y)$$

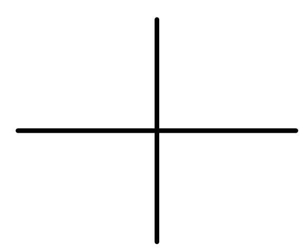
$$x = r\cos\theta$$
$$y = r\sin\theta$$

1. Write the complex number in trigonometric form.

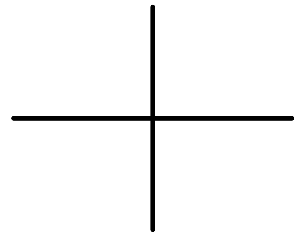
a) $5 + 5\text{i}$



b) $-2\left(1 - \sqrt{3}\text{i}\right)$

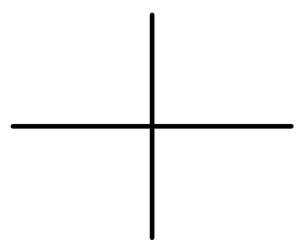


c) 7i

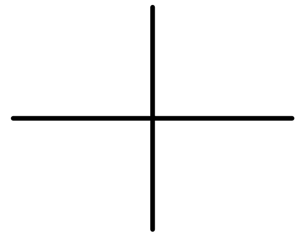


2. Write the complex number in standard form.

a) $3\cos\left(150^\circ\right)+3\sin\left(150^\circ\right)i$



b) $9\cos\frac{3\pi}{2}+9\sin\frac{3\pi}{2}$



Multiplication and Division of Complex Numbers

$$z_1 = r_1\left(\cos\theta_1 + \sin\theta_1 i\right)$$
$$z_2 = r_2\left(\cos\theta_2 + \sin\theta_2 i\right)$$

$$z_1 \bullet z_2 = r_1 \bullet r_2\left[\cos\left(\theta_1 + \theta_2\right) + \sin\left(\theta_1 + \theta_2\right) i\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos\left(\theta_1 - \theta_2\right) + \sin\left(\theta_1 - \theta_2\right) i\right]$$

3. Perform the operation.

a) $\left[3\left(\cos\frac{4\pi}{3} + \sin\frac{4\pi}{3} i\right)\right] \bullet \left[\frac{1}{2}\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4} i\right)\right]$

b) $\dfrac{32\left(\cos 150^\circ + \sin 150^\circ i\right)}{24\left(\cos 20^\circ + \sin 20^\circ i\right)}$

De Moivre's Theorem - Used to raise a complex number to the n^{th} power and to the n^{th} root

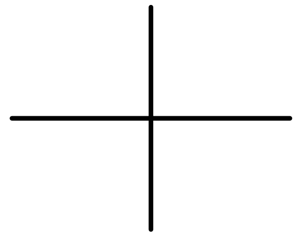
n^{th} power

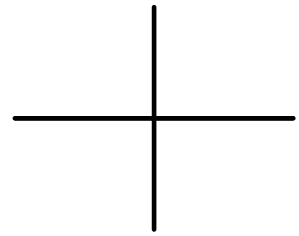
$$\left[r\left(\cos\theta + \sin\theta i\right)\right]^n = r^n\left(\cos n\bullet\theta + \sin n\bullet\theta i\right)$$

4. Use De Moivre's Theorem to find the indicated power of the complex number.

a) $\left[2\left(\cos 40^\circ + \sin 40^\circ i\right)\right]^6$

b) $\left(1-\sqrt{3}i\right)^{8}$





<u>n^{th} root</u>

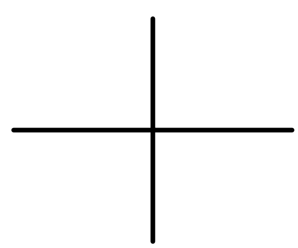
$$\sqrt[n]{\left[r\left(\cos\theta+\sin\theta i\right)\right]}$$

1^{st} Root = $\dfrac{\theta}{n}$

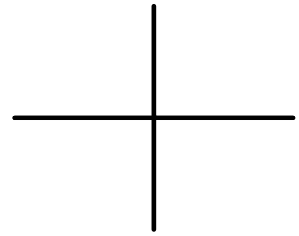
All other roots are $\dfrac{360^\circ}{n}$ apart

5. Use De Moivre's Theorem to find the indicated root of the complex number.

a) 4^{th} roots of $z=16\left(\cos\dfrac{4\pi}{3}+\sin\dfrac{4\pi}{3}i\right)$



b) 5^{th} roots of 1



c) cube roots of $2-2i$